FUNDAÇÃO INSTITUTO CAPIXABA DE PESQUISAS EM CONTABILIDADE, ECONOMIA E FINANÇAS – FUCAPE

AUREO VERDI GARCIA LEAL

BEHAVIOR AND VALUATION OF FIRMS FACING EXTREME CLIMATE EVENTS

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Mentoring: Prof. Dr. Bruno Funchal

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Behavior and valuation of firms facing extreme climate events

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COMISSÃO EXAMINADORA

Prof° Dr.: BRUNO FUNCHAL

(FUNDAÇÃO INSTITUTO CAPIXABA DE PESQUISAS EM CONTABILIDADE, ECONOMIA E FINANÇAS – FUCAPE)

Prof° Dr.: ARILTON CARLOS CAMPANHARO TEIXEIRA

(FUNDAÇÃO INSTITUTO CAPIXABA DE PESQUISAS EM CONTABILIDADE, ECONOMIA E FINANÇAS – FUCAPE)

Prof° Dr.: DANILO SOARES MONTE-MOR

(FUNDAÇÃO INSTITUTO CAPIXABA DE PESQUISAS EM CONTABILIDADE, ECONOMIA E FINANÇAS – FUCAPE)

RESUMO

A pesquisa acadêmica sobre sustentabilidade, mudança climática e eventos extremos se desenvolve nas áreas de administração de empresas, economia do meio ambiente e ciências naturais, entre outras. Estabelecendo uma ponte entre estas diferentes áreas de conhecimento, este trabalho propõe um modelo teórico para explicar o comportamento das empresas que estão expostas à mudança climática, e suas implicações para a valoração das firmas. Com base na teoria dos jogos, o modelo considera a decisão estratégica de investir em ativos para se adaptar às consequências da mudança climática. Com parâmetros específicos e uma função retorno marginal linear, mostra-se que as empresas não investirão, a menos que o mercado seja regulado e a tecnologias apropriadas estejam disponíveis. Além disso, o artigo modela o impacto de eventos extremos, decorrentes das alterações climáticas, sobre a capacidade de geração de caixa e a valorização da firma. Os resultados aplicam-se a gestores de ativos, legisladores, investidores, gerentes de empresas e pesquisadores.

Palavras-chave: mudança climática; comportamento empresarial; valoração de empresas

ABSTRACT

The academic research on sustainability, climate change, and extreme events develops in business administration, environmental economics, and natural science areas, among others. Bridging these different bodies of knowledge, this paper proposes a theoretical model to explain the behavior of the firms that stand before climate change and its implications for the firm´s valuation. Building on game theory, the model considers the strategic decision of investing in assets to adapt to the consequences of the climate change. With specific parameters and a marginal linear payoff function, it is shown that firms shall not invest, unless the market is regulated and appropriate technologies are available. Moreover, the paper discusses the impact of extreme events, arising from climate change, over the cash generation capability and the valuation of the firm. The results hold for asset managers, policy makers, investors, corporate managers, and researchers.

Keywords: climate change; firms´ behavior; firm´s valuation

SUMMARY

1. INTRODUCTION

Beyond the *locus* where transformations of energy, mass, and life occur, the environment is the system in which the economic activities take place. The system receives energy input in the form of solar radiation, which is the only external source that drives every transformation, including the economic cycle of production and consumption (PERMAN et al., 2003). The Intergovernmental Panel on Climate Change (IPCC) evidences the inherent relationship and interdependence between economy and environment, especially as it holds economic activities responsible, at least partially, for the global warming witnessed in recent years (PACHECO et al., 2014).

The problem aggravates when governments, firms, and consumers do not agree on the severity of the global warming problem, despite the scientific consensus about the anthropogenic responsibility, and the warning signals about severe climate change (CC henceforth) consequences (PACHECO et al., 2014). However, as observed by Dean and McMullen (2007), the environmental economics indicates that the very nature of the problem is largely due to market failures inherent in the economic system, which prevent resolving the problems and often motivate environmentally degrading entrepreneurial behaviors.

There are contradictory messages sent by leaders, Pope Francis (2015) has urged public attention to the environmental situation, when his recent encyclical stated that the youth are questioning how anyone can build a better and sustainable future, without thinking of the environmental crisis. However, at the same time, fossil fuel subsidies are still common in many countries, while the re-allocation of such resources could provide the necessary financial support for universal health coverage in these countries (GUPTA et al., 2015).

The CC consequences, such as increased occurrence of heat waves and droughts, start to be a real threaten to the firms. In 2014, Coca-Cola´s representative declared that the company recognizes events related to climate as a threat to the supply of basic production inputs, such as water, sugar, and fruits (DAVENPORT, 2014). Caldecott (2011) questions rather the continuous investment in high carbon economy that could not launch the fundamentals for the next sub-prime crisis, once 80% of the known fossil fuel reserves, declared by energy and mining companies, may become stringed assets, provided the nations live up to their commitments to control global temperature (CALDECOTT, 2011).

Goodall (2008) noticed the absence of themes, such as global warming and CC, in top business and management journals. This situation and the interdisciplinary character of sustainability suggest the need for a framework in which global warming and CC may be addressed in a way to integrate different aspects of the problem and to build bridges in order to facilitate the dialogue among practitioners and academics from different bodies of knowledge.

Apparently, such dialogue among different expertise has not been the reality in the academy. Research in the business domain has focused on the correlation between firms' performance and sustainability (PAINE, 2004; ECCLES et al., 2011; BROCHET et al., 2013), as well as the utilization of sustainability as a marketing and strategic tool (VARADARAJAN, 1992; MENON & MENON, 1997; PEATTIE, 2001; MCELHANEY, 2009; MICHELON, 2011). Research in the economy domain contributes with extensive discussion on the mechanisms to enforce compliance, the ethical perspectives, and economic development theory (BARRET, 1994; ONU/WIDER, 1997; STERN, 2004; SACHS, 2009). Natural sciences research and environmental global institutions propose modelling techniques and stress the sense of urgency for action (DIETZ, 2007; OECD, 2009; WALSH, 2011; PRESTON, 2012). This paper is intended to contribute to bridge these different bodies of knowledge, namely, business administration, environmental economy, and natural science.

In order to contribute to this discussion, the first objective is to propose a theoretical model to explain the behavior of economic agents that stand before the circumstances of a changing climate. As an example, seaports may need to invest in coastal structures due to the rise of sea level, industrial sectors may invest in alternative production processes due to shortage of water fonts, and carbon dependent activities may need to find technological alternatives for energy sourcing. Building on game theory, the first objective is to explain the decision process and the conditions for the firms to invest in adaptation to the new environmental outlook.

The second objective is to estimate the impact of the decisions related to adaptation to CC over the value of the firms. In order to accomplish this task, a hypothetical situation is introduced, in which the aggravation of the climate disturbs ends up to create a situation of distress. Under this circumstance, the cash generation capability of the firms is compromised, which the market may anticipate, affecting its valuation. Building on discounted cash flow method, the second objective is to model the valuation of firms that stand before climate change impacts.

The first contribution of this paper is to bring to the domain of business and management research, a framework that is only usual in economics and natural science. The goal is to launch a common ground of discussion to integrate business and environmental economy researches.

As a second contribution, the paper models the strategic decision of adaptation to CC from the investment standpoint, while typically economy literature utilizes the pollution flow as the key variable for modelling. This different approach widens the discussion, while the results remain consistent with previous research.

A third one is the support provided by this model to recent discussion and movements in the market directed to diminishing the exposure to fossil fuel related businesses from the fund's endowment, among institutional investors.

After a review of the literature (section 2), where fundamental concepts are discussed, the proposed model is developed in the next two sections. Section 3 proposes a game, played by two firms, to explain the relationship among the economic agents and the behavior of the firms. This section ends with the determination of the firms´ profits at equilibrium. Section 4 proposes two scenarios, “*business as usual”* and “*extreme events*,” in order to determine the valuation of the firms, when exposed to climate change and eventual extreme events (EE), that may affect their capability to generate cash.

Next, section 5 discusses the driving forces of the firms’ behavior and the implications for their valuation, as well as the direct relationship with very recent public news. The paper concludes, in section 6, with its limitations and suggestions for future research.

2. LITERATURE REVIEW

In this section, I review fundamental concepts to establish the foundation for the development of the proposed model, in the next section.

2.1. SUSTAINABILITY IN DIFFERENT DOMAINS

In the business domain, explicit notions of ethical consumption emerged in the 1970’s, and evolved along the years, up to the current concept that a sustainable production and consumption should perpetuate today’s material standards of living for future generations (PEATTIE, 2001; SACHS, 2009).

The broad implications of this concept for management practices and theory, regardless of business leaders’ fundamental beliefs, may not be ignored and affect virtually all areas of management. From relationship with stakeholder, firm´s valuation, risk management, marketing, product design, production, up to logistics, the entire value creation chain is affected (CARROLL, 1991; VARADARAJAN, 1992; PEATTIE , 2001; KUNREUTHER, 2004; BRINKMAN et al., 2008; THE GENEVA ASSOCIATION, 2009; SPRINKLE & MAINES, 2010; CONNELLY et al., 2011; ECCLES et al., 2011; ZHAO et al., 2014).

From the environmental science, sustainability intimately relates to the recognition of a range of services provided by the natural environment that are of fundamental importance to human well-being, health, livelihoods (COSTANZA, et al., 2006; SHAW & WLODARZ, 2013; COSTANZA et al., 2014). Such services delivered by the environment to the economic system are goods for consumption, input for production, receptacle of waste, and location (PERMAN et al., 2003). The eventual loss of these services as consequence of CC and over utilization led to the development of complex mathematical models to simulate and estimate the impacts (IPCC, 2014). When environmental scientists realized the seriousness of the treats, adaptation, adaptive capacity, vulnerability, resilience, exposure and sensitivity became integral part of the sustainability concept and have wide application to CC research (SMIT & WANDEL, 2006)

In the economy domain, a long-term growth process that results from a virtuous cycle of savings and investment, which in turn increases employment and income, is a traditional conception of “sustained growth” (ROMEIRO, 2012). Under this concept, the economic development of nations is sustainable and socially desirable, when it is efficient or inclusive. However, the emergence of the global warming issue and the depletion of natural resources introduced additional constraints into the debate about economic sustainable development.

According to Romeiro (2012), the so-called “*zeroists*” or (pejoratively) “neo-Malthusians,” environmental limits would lead to catastrophes if economic growth does not stop. In the other extreme of the debate, others postulate that technology expands the economic efficiency, in such way, that may virtually eliminate the scarcity of natural resources, and capital and innovation would largely replace natural resources.

The latter sees the environmental crisis as a market failure, with natural resources treated as public goods, thus generating a negative externality problem. In this sense, government action is necessary to correct this market failure by creating the conditions for economic agents to “internalize” the costs of the degradation they cause. Thus, a price tag for natural resources would be an alternative to provide the necessary conditions for sustainability.

Research from Neumayer (2000) summarizes the foundation for some optimism by considering that the economic system reaches new equilibrium, when changes occur in the system. There shall be substitution of the resource that became scarce and expensive by another more abundant resource. The system may also offer new products that use different resources and a rise in the price may lead to recycling.

Finally, capital can replace natural resources and technical progress increases the efficiency of resource utilization (NEUMAYER, 2000).

2.2. ENVIRONMENTAL QUALITY

The concept of private and public goods is associated with the ownership and control of the goods. Individuals own private goods, there is competition for using the goods, and it is possible to exclude potential users, once individuals retain property rights. By definition, the utilization of a public good by someone does not impede someone else from using it, and does not reduce availability of the public goods. Consequently, there is no rivalry in use and property rights cannot be attributed to individuals. Under certain circumstances, an exclusion may even be possible but not desirable due to normative reasons, for instance the access to a public beach.

Formally, for the pure public good A, U_l^A denotes the quality U of a public good l used by individual A, thus:

$$U_l^A = U_l^B = \cdots = U_l^Z = U_l \qquad (1)$$

When N denotes the number of users of a certain service l and $\overline{N}$ the capacity limit of the system. Above this limit, there exists a congestion problem and the derivate of the quality function is negative (SIEBERT, 1981). If the system is not able to replace the resources timely, the overall situation becomes unsustainable (SIEBERT, 1981). Mathematically, it may be represented as follows:

$$\frac{\partial U_l^A}{\partial N} = \frac{\partial U_l^B}{\partial N} = \frac{\partial U_l^C}{\partial N} = \cdots = \frac{\partial U_l}{\partial N} \begin{cases} < 0 \; for \; N > \overline{N} \\ = 0 \; for \; N \leq \; \overline{N} \end{cases} \qquad (2)$$

The negative derivate expresses the degradation of the services level provided by environment, which the depletion of the natural resources and loss of environmental quality.

Siebert (1981) shows that the environmental and economic systems have a sustainable relationship when a set of rules and behaviors is able to restrict the utilization up to the capacity limit, allowing the system to regenerate itself, and continue to deliver the services. For example, there is a maximum number of persons visiting a park, above which nature is not able to regenerate itself, and the quality of the park declines.

In other words, the capacity to delivery environmental services is finite, and the quality declines when it reaches the capacity limit of support, as mathematically described before, and, at this point, a problem of congestion emerges (PERMAN et al., 2003). Consequently, some sort of regulation or self-enforcement must exist in order to curb the exhaustion of the resources.

Figure 1 depicts the relationship between environmental and economic systems.

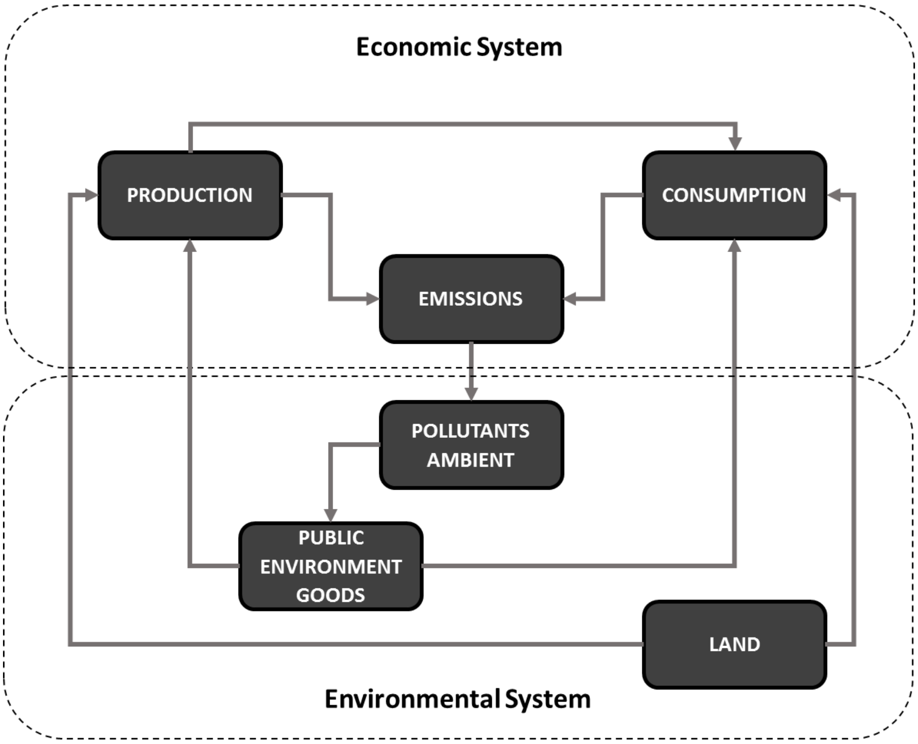

Figure 1 - Environmental and economic systems
(Source: Adapted from Siebert, 1981)

2.3. EXTERNALITIES

An externality exists if the output of an activity depends on the output or on the inputs of another activity. When the output of the dependent activity increases with the increase of the output (or input) of the independent activity, there is a positive externality, otherwise, it is negative. Mathematically:

$$Q_i = f_i(R_i; R_j, Q_j) \quad where \ \frac{\partial Q_i}{\partial Q_j} \neq 0 \ or \ \frac{\partial Q_i}{\partial R_j} \neq 0 \qquad (3)$$

Where i and j are two products in an economic set, R denotes production inputs and Q production outputs (SIEBERT, 1981).

Thus, an externality occurs when the production or consumption decisions of one agent have an impact on the utility of another agent in an unexpected way, and there is no compensation or payment by the generator of the impact to the affected party (PERMAN et al., 2003). Some pollution or emissions are typically undesirable sub-products that normally come out of the production processes.

A classic example of externality is the steel plant by a river, upstream, which produces some sort of pollutant. From the fishery standpoint, which is located by the same river, downstream, the pollutant released in the river by the plant upstream is a negative externality, once the fishery output decreases when the output from the steel plant increases. It is also said that the costs associated with the release of pollutants are “social costs,” not internalized by the steel plant, but shared with the society.

Therefore, externalities are “non-market” interdependencies between economic activities. The environment is one possible system to establish such interdependency. When the market mechanisms are not able to mediate the relationships between economic agents, the phenomenon of externalities is present and the market fails to induce the optimal conditions (PERMAN et al., 2003). On the other hand, since the 90´s, a number of researchers have related the deterioration of the environmental system to inappropriate government policies and poverty, and have given less importance to the market failure (DASGUPTA & MÄLER, 1996).

Government intervention offers the possibility of realizing efficiency gains, by eliminating or mitigating situations of market failure. One way is the development of institutional arrangements for establishing and supporting property rights. Fiscal instruments is another way for government to act. By fiscal instrument, it is understood tax, subsidies, and marketable permits. In addition, government can nurture the availability of information and technology, reducing the uncertainties associated with CC and increasing the efficiency of solutions (DASGUPTA & MÄLER, 1996; PERMAN et al., 2003; ROMEIRO, 2012).

This paper explores the externality situation, with two firms and government exerting the regulatory function over pollutant emissions, and technology is addressed as an important variable in the model.

2.4. MANAGERIAL ROLE

As global warming becomes more salient, among other climate signals interpreted by science, new regulations, technological innovations, and change in consumer behavior may significantly affect the valuation of firms (BRINKMAN, HOFFMAN, & OPPENHEIM, 2008). The capacity of reaction may not be as quick as the urgency requires, not only due to rational and economic reasons, but also due to the psychological distance perceived by management.

Researchers have identified an association of the challenge of CC with impacts that are geographically and temporally distant (SPENCE, POORTINGA, & PIDGEON, 2012). Despite of this psychological distance, and the fact that the response to the climate threat has a global amplitude, yet it requires local actions, mobilization, and regulation.

Local actions require, in turn, commitment of consumers, private sector, and regulation by the government. All these agents have important role in this process. (ADGER, ARNELLA, & TOMPKINS, 2005).

At firm´s level, investors and top managers play a fundamental role. The upper echelons theory proposes that the organizational outcomes arise largely from the decisions of a dominant coalition, and cognitive bases and values of this group influence their decisions (CONNELLY et al., 2011). Thus, there exists an important behavioral component in the decision process, which varies among the organizations, especially when dealing with such complex and unparalleled subject.

2.5. MITIGATION AND ADAPTATION

Traditionally, there are two categories of responses to CC: mitigation and adaptation. The first category refers to projects to reduce greenhouse gas (GHG) emissions. Typically, they are global actions, aiming at the stabilization of GHG concentration in the atmosphere and consequently avoiding interference with the climate system. Examples are the substitution of fossil fuels based energy by renewables sources, reduction of deforestation, and innovative alternatives for transportation (THE GENEVA ASSOCIATION, 2009).

However, due to the inertia of the climate system practical results shall happen in the long term, regardless the actions taken for mitigation, which brings to the discussion the need for investment in adaptation assets, the second category of responses. The IPCC´s Fourth Assessment Report (AR4) defines adaptation as “adjustments in natural and human systems in response to actual or expected climate stimuli or their effects” (IPCC, 2007).

Adaptive responses may be reactive or anticipatory, autonomous and planned, substitutes or complements. For instance, once a flood occurred, government may launch programs to protect the river and avoid future occurrences, as a reactive response. However, these programs could have been launched prior to the floods, by anticipation. Anticipation requires foresight and planning. On the other hand, in a cost-benefit analysis process, an investment should be delayed as long as the benefits of delay (avoided investment costs) are lesser than the associated costs (higher CC damages or taxation) (FRANKHAUSER, SMITH, & TOL, 1999).

The continuous process of corporate risk management and governance could be able to manage the adaptation process; however, it would require anticipation of the impacts and its direction. Such information is typically not available, especially at the local level, where the company's assets, operations, and employees may be affected (WEST & BIANCHI, 2013). For instance, the level of rainfall is expected to change, but it may not be clear whether it shall increase or decrease in certain areas (THE GENEVA ASSOCIATION, 2009). Under this uncertainty, increasing the flexibility and robustness of systems to function under a wider range of climatic conditions is another important alternative for adaptive response (FRANKHAUSER, SMITH, & TOL, 1999). Therefore, this is another example of adaptation: investment in flexibility and robustness.

Consequently, different alternatives of investment and strategies involve different costs and produce different results. In order to emulate this concept, this paper considers a variable e to denote the technological efficiency of the different alternatives. The better is the efficiency of the technology; the bigger shall the reduction of emissions.

In summary, CC posts a challenge for policy and decision makers, given the ambiguity and uncertainty of the likelihood of occurrence of climate related events, their potential impacts, as well as the lack of information and previous experience, and the strong dependence on management individual perceptions. Mitigation brings short-term results, relatively easier to justify economically, while adaptation assets are riskier, and its approval faces stronger resistance.

2.6. VALUATION UNDER UNCERTAINTY

From theory of probability, weather extremes may change much faster than weather means. According to this basic principle, the society shall notice weather extremes, such as high and low temperatures, or draughts and floods, earlier than any consensus about changes in mean climate may be achieved (FRANKHAUSER, SMITH, & TOL, 1999).

For the proposed model, there is an uncertainty related to the possibility of extreme events (EE), whose risk profile characterizes by low probability and high consequences (LP-HC) (KUNREUTHER, 2004). For example, an earthquake may directly affect a homeowner´s property, or indirectly, when a damage over neighbor´s property cause collateral damages (FRANKHAUSER, SMITH, & TOL, 1999).

In addition, certain assets may suffer from unanticipated or premature write-downs, or conversion to liabilities, especially if governments live up to their commitments to keep global warming below 2 degrees. These are called “stranded assets.” As much as EE, stranded assets may cause cessation or diminution of cash generation (BLOOMBERG Financial LP, 2013). Consequently, the EE and stranded assets affect the capability of the firms to generate the same cash flows as prior to the EE.

The discounted cash flow (DCF) model has been a traditional method to value assets (GORDON, 1959). The DCF method forecasts cash flows for an initial set of years, so called explicitly modeled years. Additionally, the method assumes a constant growth, into the indefinite future, however, this paper challenges this assumption, once it intends to explain the impact of extreme events and stranded assets, as discussed before, over the value of the firms exposed to the CC.

According to Chen (1967), there are two models to the problem of asset valuation under uncertainty. The certainty-equivalent method converts each future cash flow to its certainty equivalent and discounts at an appropriate rate. The second one, risk-adjusted method, utilizes the discount rate to express the risk uncertainty and the time value of the money, and discounts each future cash flow at a risk-adjusted discount rate (CHEN, 1967).

Damodaran (2002; 2006) highlights the possibility to account for the distress by creating all possible scenarios, ranging from the most optimistic to the most pessimistic. For each scenario and each period, assign a probability and a cash flow. Thus, the expected cash flow is $\sum_{j=1}^{j=n} \epsilon_{jt}.CF_{jt}$, where ϵ_{jt} and CF_{jt} are the probability and the cash flow under of scenario j in period t. The consideration of only two scenarios, "*on going concern"* and "*distress*," may be enough for simplification purposes (DAMODARAN A. , 2002; DAMODARAN A. , 2006).

Following a similar rational, Saha and Makiel proposed the discount factor in equation (4) in order to account for a distress situation, with probability ϵ and a diminution of cash generation cash of δ. With this approach, ϵ represents the probability of occurrence of an event that disturbs firms´ operations, causing a cash flow reduction by a percentage δ (SAHA & MALKIEL, 2012):

$$DF = \sum_{t=1}^{\infty} \left[\left((1-\epsilon) + \epsilon(1-\delta) \right) \frac{(1+g)}{(1+r)} \right]^t = \frac{(1-\epsilon.\delta).(1+g)}{\left(r - g + \epsilon.\delta.(1+g) \right)} \quad (4)$$

3. ASSUMPTIONS

The terms "payoff function" and "profit function" apply indistinctively and are the mathematical representation of the benefit expected by the shareholder in order to compensate for the enterprise venture. The equation $\pi(q) = p(q).q - c(q)$ often represents a general payoff function, where q is quantity, p is product price, c is marginal cost of production, and is assumed strictly concave and twice differentiable in q (TIROLE, 1988).

In economic theory, profits and production functions are usually assumed to be concave, that is, production outputs increase with more resources (inputs or factors of production), however the increment rate becomes smaller, as the resources increases, at least in the short run. It is often called "law" of diminishing marginal product. The interpretation is straight forward.

On the other hand, at a given technology, the quantity of pollution or emissions, released by the firms, increases proportionally or progressively with the production output. Without loss of generality, let's assume one type of pollutant generated by the firms, and the function S represents the emissions as dependent of the output (joint-products of the production process).

Let $S_i = s(q_i), \forall i$ be the function that describes the emissions caused by the quantities produced by each firm F_i and $S = s(q_1) + s(q_2)$ the total amount of emissions released in nature.

According to Siebert (1981), if one applies the mass-balance concept to the production function, the emission function as introduced above is proven to be convex, that is, emissions increase with more production (outputs), up to a point that any additional output may increase dramatically the emissions.

The interpretation may not be intuitive. The principle of conservation of mass states that the mass that enters a system must either leave the system or accumulate within the system. Without loss of generality, if one admits no accumulation, then the total mass of inputs must equal the total mass of outputs, which includes the joint-products, i.e., the emissions. Therefore, if the production function is concave, the pollution or emissions function must be convex in order to respect the principle.

Suppose there are compelling reasons, further detailed, for the firms to start to invest in mitigation or adaptation to CC at the period t_n. Firms shall build appropriate assets to protect themselves against eventual foreseeable impacts and there are technologies available to build such adaptation assets. Let´s say that firms start to CAPEX, at the period t_n, as shown in Figure 2.

In addition, expectations of weather events evolve over time, from a very low probability ϵ, up to a moment when the signals are strong enough so that the occurrence of extreme events is significantly high. Let´s represent the moment when extreme events become a real threat by t_m, as shown in Figure 2 as well. This concept will be further discussed, for valuation of the firms under distress

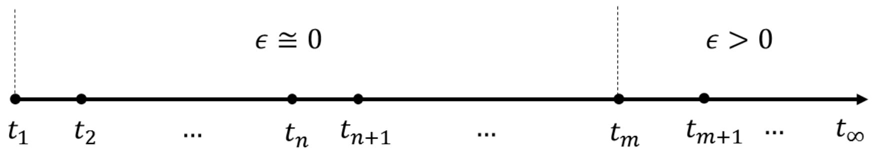

Figure 2 – Timeline and expectation of extreme events

Assume that the capital expenditure (CAPEX) is proportional to the emissions produced by each firm. As emissions are function of production, thus the bigger is the production, the bigger shall be the CAPEX needed to adapt to the CC. Furthermore, consider a linear marginal cost function to describe the relationship between CAPEX (K) and quantities:

$$K = k.q^2, \quad k > 0 \tag{5}$$

It is clear that, when a firm invests in adaptation assets, the emissions must decrease. The rate of abatement must be proportional to the efficiency e of the technology in place. The better is the technology of adaptation; the bigger shall be the abatement of the emissions. Therefore, the function below denotes the emissions of each firm, before and after the process of adaptation:

$$S = s(q) = \begin{cases} s.q^2, & \text{without CAPEX in adaptaion} \\ s.q^2 - e.(k.q^2), & \text{with CAPEX} \end{cases} \qquad e, k, s \neq 0 \tag{6}$$

Figure 3 depicts the behavior of the emission function, with CAPEX in place, and how it would have been if no adaptation had started.

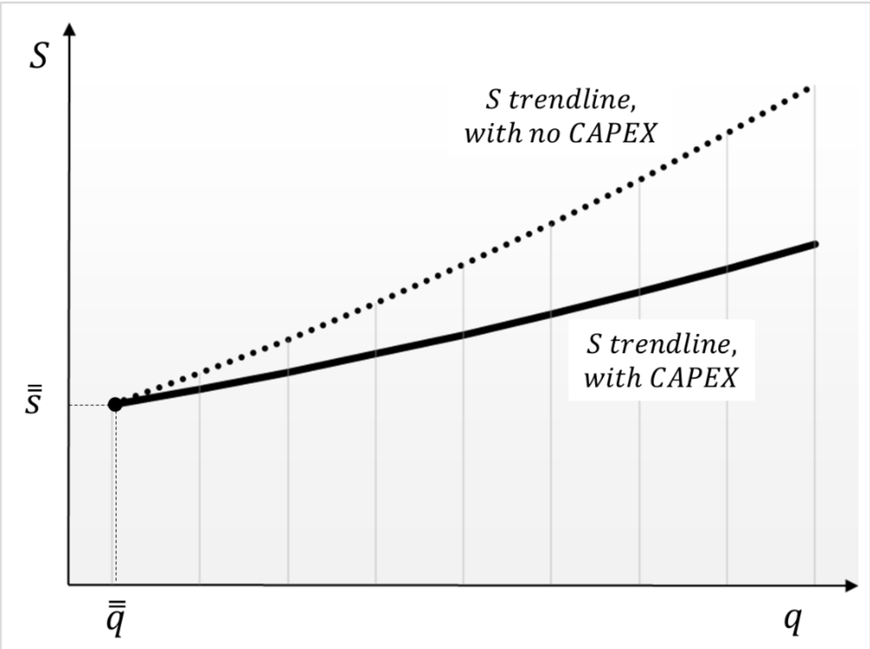

Figure 3 – Trendline of the emission function

Let T denote the regulatory function performed by the government. The extent the government enforces regulatory constraints is proportional to the emissions. As emissions are proportional to the quantities produced, the proposed model assumes a constant marginal function for the regulatory function, proportional to the damage caused by the firm, thus:

$T = t.s(q), \quad t > 0$	(7)

The model assumes two firms (F_1, F_2). They are identical, have the same marginal cost $c,\ c > 0$, the same cost of capital $r,\ r > 0$, and the inverse demand function given by:

$p = a - q_i - q_j, \qquad \forall\ i, j$	(8)

Where the (i, j) denotes each of the firms. Let the profit function be:

$\pi_i(q_i) = [p - c].q_i - r.K_i - T_i\ ,\ \ \forall\ i$	(9)
Where:	p *is the product unit price* c *is the marginal cost* q_i *is the quantity produced by* F_i r *is the cost of capital* K_i *is the CAPEX in adaptation by* F_i T_i *is the regulatory function exerted by the government*

The original profit function (9), in accordance to (5), (6), (7), and (8), rewrites now as equation (10):

$$\pi_i(q_i, q_j) = [(a - c) - q_i - q_j]q_i - rkq_i^2 - t(\bar{\bar{s}} - ek)q_i^2, \quad \forall i \tag{10}$$

Following the assumption of a declining marginal returns, it is a concave function, twice differentiable in q, and the conditions for profit maximization are:

$$\max_{q_i} \pi_i(q_i, q_j) \iff \frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = 0, \quad \frac{\partial^2 \pi_i(q_i, q_j)}{\partial^2 q_i} < 0, \ \forall i \tag{11}$$

The last two terms of the profit function (10) denotes the total cost bear by the firms due to externalities, being rk the cost of the CAPEX and $t(s - ek)$ the net cost, arising from regulation. Herein, it will be noted as $\omega = rk + t(s - ek),\ \omega \geq 0$. After some rearrangements:

$$\omega = ts - rk\left(\frac{te}{r} - 1\right) \geq 0 \tag{12}$$

The development of these conditions leads to the quantity that maximizes the profit, and the respective profits, as shown by the equations below. This formulation is extensively discussed in Appendix 1.

$$q_i^* = \frac{(a - c)(1 + 2\omega_i)}{[4(1 + \omega_i)(1 + \omega_j) - 1]} \ and \ q_j^* = \frac{(a - c)(1 + 2\omega_j)}{[4(1 + \omega_i)(1 + \omega_j) - 1]}, \forall i$$

$$\pi_i = \frac{(a - c)^2(1 + \omega_i)(1 + 2\omega_j)^2}{[4(1 + \omega_i)(1 + \omega_j) - 1]^2}, \quad \pi_j = \frac{(a - c)^2(1 + \omega_j)(1 + 2\omega_i)^2}{[4(1 + \omega_i)(1 + \omega_j) - 1]^2}$$

In summary, firms choose their own production quantities and decide whether to adapt, or not. Therefore, for the model, the only endogenous variable is the production quantity.

Table 1 depicts the list of variables.

Exogenous	a	*coefficient of inverse demand function*
	c	*marginal cost*
	t	*tax bracket*
	t_n	*time fence when governments starts regulation*
	t_m	*time fence when EE occurs*
	ϵ	*probability of occurrence of EE*
	δ	*expected cash flow loss due to EE*
	r	*cost of capital*
	g	*cash flow growth*
	e	*efficiency of technology*
	k	*slope of CAPEX function*
	s	*slope of emission function*
Endogenous	q	*production quantity*

Table 1 - Endogenous and exogenous variables

4. THE MODEL

Next two sections introduce the framework to explain the behavior of the firms under the situation of CC, as well as the valuation of such firms, considering a chance of occurrence of extreme events (distress).

4.1. FIRM´S BEHAVIOR

Game theory and Cournot model are the foundations of the proposed model. They shall represent strategic and economic decisions of interdependent agents. The model considers a game that takes one-period interaction, simultaneously. We began with a duopoly, without loss of generality. In addition, this is a static and complete information game with two oligopolistic firms ($\boldsymbol{F_1}, \boldsymbol{F_2}$), operationally identical, producing a homogeneous product. The firms operate under a government.

4.1.1. Strategic Choices

For the extensive-form representation of the game, refer to Figure 4.

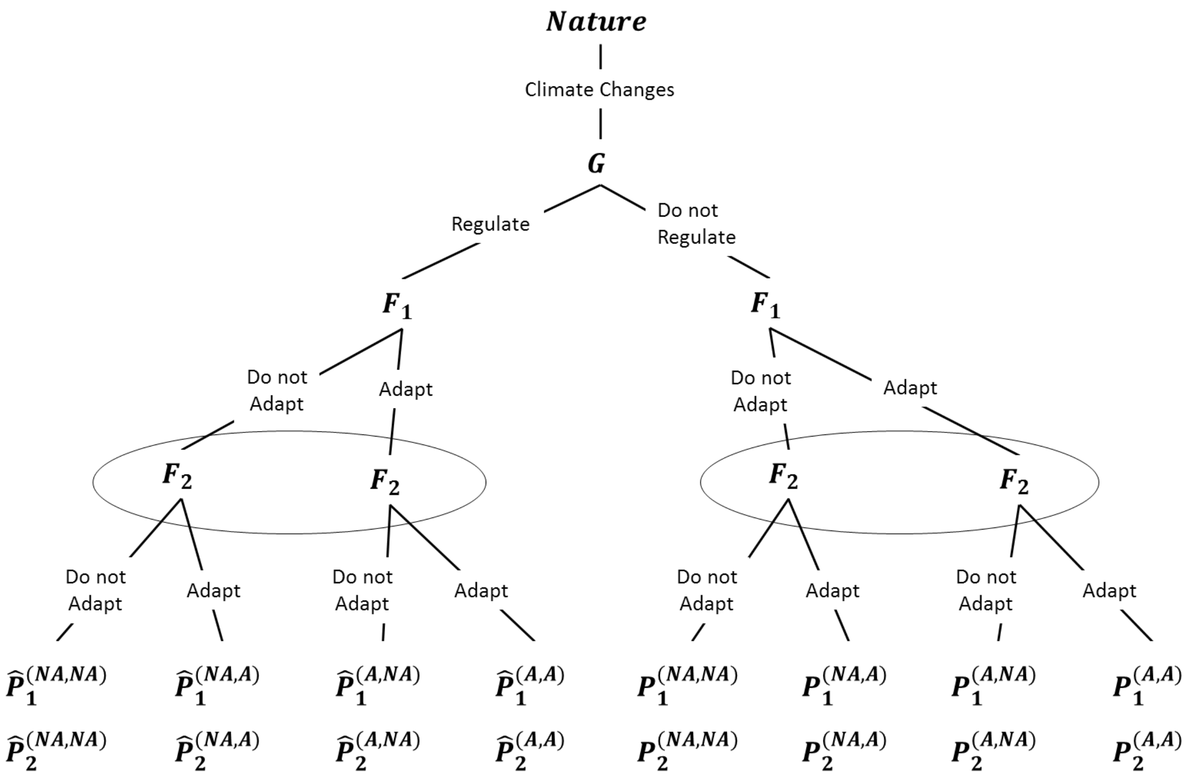

Figure 4 - Extensive-form representation of the game

In a first moment, nature signals with a change. The government G may (or may not) regulate the market, corresponding to the left and right hands of the tree, respectively.

The firms may decide to adapt (*A*) or not adapt (*NA*), which generates eight different combination pairs of payoffs. Let the pairs (A, A), (NA, NA), (A, NA), and (A, A)

denote the combination set of decisions adopted by (F_1,F_2), where A means a choice for investing in adaptation and NA, otherwise.

Following this notation, in the right branch, $P_1^{(A,A)}$ denotes the payoff of the firm F_1 when both firms decide to adapt, while $P_1^{(NA,NA)}$ denotes the payoff of the firm F_1 when both firms decide otherwise. In addition, $P_1^{(A,NA)}$ denotes the payoff of the firm F_1 when F_1 chooses to adapt and F_2 otherwise. Finally, $P_1^{(NA,A)}$ is the payoff of the firm F_1 when F_1 chooses not to adapt and F_2 otherwise. The same holds for F_2. For the left branch, an accent letter $\hat{P}$ notates the payoff for its four pairs of combinations, likewise previously described.

4.1.2. Equilibrium

The intention of this section is to explain the behavior of the firms, by the equilibrium condition of the game, above described. According to economic theory, a partial equilibrium occurs when, for a given price, markets are cleared, and each producer maximizes his profit subject to the production function.

Following a Cournot duopoly model, firms determine, independently and simultaneously, their quantities supply $q_i, \forall i$ to maximize their own payoffs, following the principle of a profit maximizing and rational firm. Furthermore, the decision to invest in adaptation assets is constrained by the payoff of the investment.

Considering $P_1^{(NA,NA)}$ as the baseline, the paper compares the payoff´s resultant from the independent decision of each player to adapt or not and shows there no reason to adapt, given the government does not regulate (right branch).

In a second step, considering $\hat{P}_1^{(NA,NA)}$ as the new baseline, after government´s regulation, the same comparison of the payoff´s resultant from the independent decision of each player is performed and it shows that, under a regulated market, there reasons to adapt.

These comparisons of payoffs, in each combination set, leads to the propositions drawn next. See Appendix 1 for details.

Proposition 1: in a duopolistic market, when government does not regulate, firms shall not internalize the externality.

This proposition is the theoretical prediction of the situation in which government does not utilize mechanisms of incentive and the equilibrium is established when firms do not adapt (NA, NA), since their payoff shall decrease, with such an investment, regardless the cost of capital, size of the investment, or technology available.

Firms do not adapt because its payoff decreases, regardless the other firm´s decision, as interpreted by expression P1:

$\frac{P_1^{(A,NA)}}{P_1^{(NA,NA)}} < 1$	P1

Corollary 1: if a firm decided to internalize the externality, created by a CC, the payoff of the other firm would increase.

$\frac{P_2^{(A,NA)}}{P_2^{(NA,NA)}} > 1$	C1

Corollary 2: when both firms move towards adaption, the payoffs decrease for both of them.

$\frac{P_1^{(A,A)}}{P_1^{(NA,NA)}} = \frac{P_2^{(A,A)}}{P_2^{(NA,NA)}} < 1$	C2

Proposition 2: in a duopolistic market, when government does regulate, with the enforcement of taxation, firms shall internalize the externality subject to relationship between the tax bracket, efficiency of the technology and cost of capital.

This proposition explains the situation in which government utilizes taxation as regulatory mechanism to establish a price on the externality. Firms are then encouraged to invest in adaptation. As discussed later, the decision to move ahead with the investment depends whether the efficiency of the technology and the cost of the investment can offset the tax imposed by the government. Further, it will be possible to show the relationship between tax and technology.

With the enforcement of taxation, firms shall adapt once its payoff increases by investing in adaptation, and moving from the trivial position (NA, NA) to (A, NA), regardless the other firm´s decision, as interpreted by expression P2:

$\frac{\hat{P}_1^{(A,NA)}}{\hat{P}_1^{(NA,NA)}} > 1$	P2

Corollary 3: if a firm decided to adapt, the payoff of the other firm would decrease.

$\frac{\hat{P}_2^{(A,NA)}}{\hat{P}_2^{(NA,NA)}} < 1$	C3

Corollary 4: a new equilibrium shall establish when both firms decide to invest in adaptation and both payoffs increases.

$\frac{\hat{P}_1^{(A,A)}}{\hat{P}_1^{(NA,NA)}} = \frac{\hat{P}_2^{(A,A)}}{\hat{P}_2^{(NA,NA)}} > 1$	C4

Corollary 5: in a duopolistic market, when the firms internalize the externality created by a CC, the total production decreases, which means a Pareto improvement. From economic theory, when a negative externality exists, the market does not incorporate the additional cost, thus, too many goods will be produced. When internalizing that cost, less products will be produced.

$\frac{\hat{q}_1^{(A,A)} + \hat{q}_2^{(A,A)}}{q_1^{(NA,NA)} + q_2^{(NA,NA)}} < 1$	C5

4.2. FIRM´S VALUATION

The intention of this section is to discuss the implications of the firm´s behavior, over their valuation. The model proposes that an EE posts a significant threat, at period t_m and discusses the impact over the value of the firms exposed to such possibility.

4.2.1. Scenario Business as Usual

The paper creates two scenarios in order to model the influence of the climate change over the valuation of firms. The first one, a scenario used as benchmark, is “business as usual” (BAU) and no climate disturbance is supposed to exist. The value of the firms is the present value (PV) of the expected future cash flow, which is an application of trivial concepts from finance. Figure 5 depicts the scenario BAU, as well as respective the payoff and PV formulation. Let PV^{BAU} denote the PV under scenario BAU.

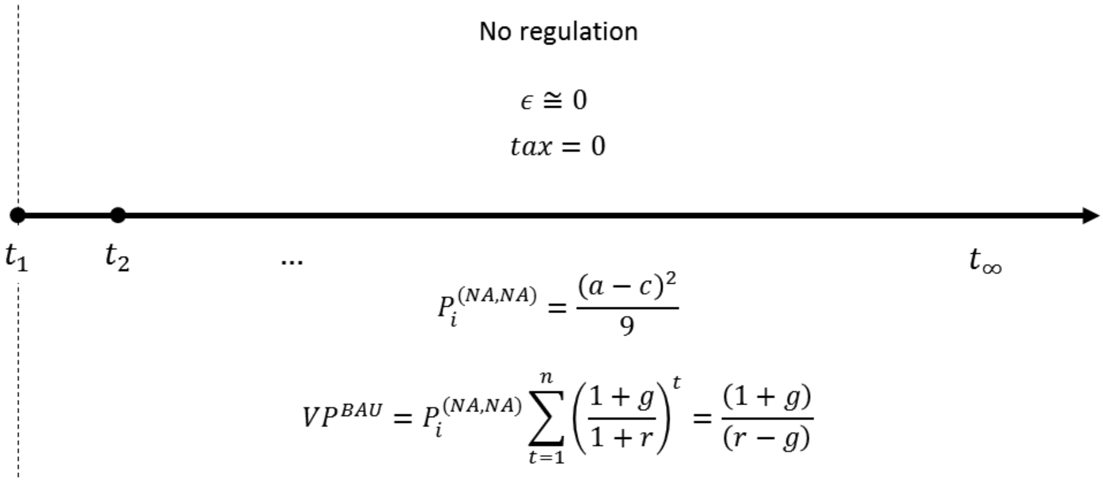

Figure 5 - Scenario BAU

4.2.2. Scenario Extreme Events

The second scenario is the study case, referred as scenario EE. Building this scenario requires a description of a hypothetical future. Figure 6 depicts the timeline of some hypothetical events related to CC, with time fences $t_0, t_n,$ and t_m, as well as three distinct phases.

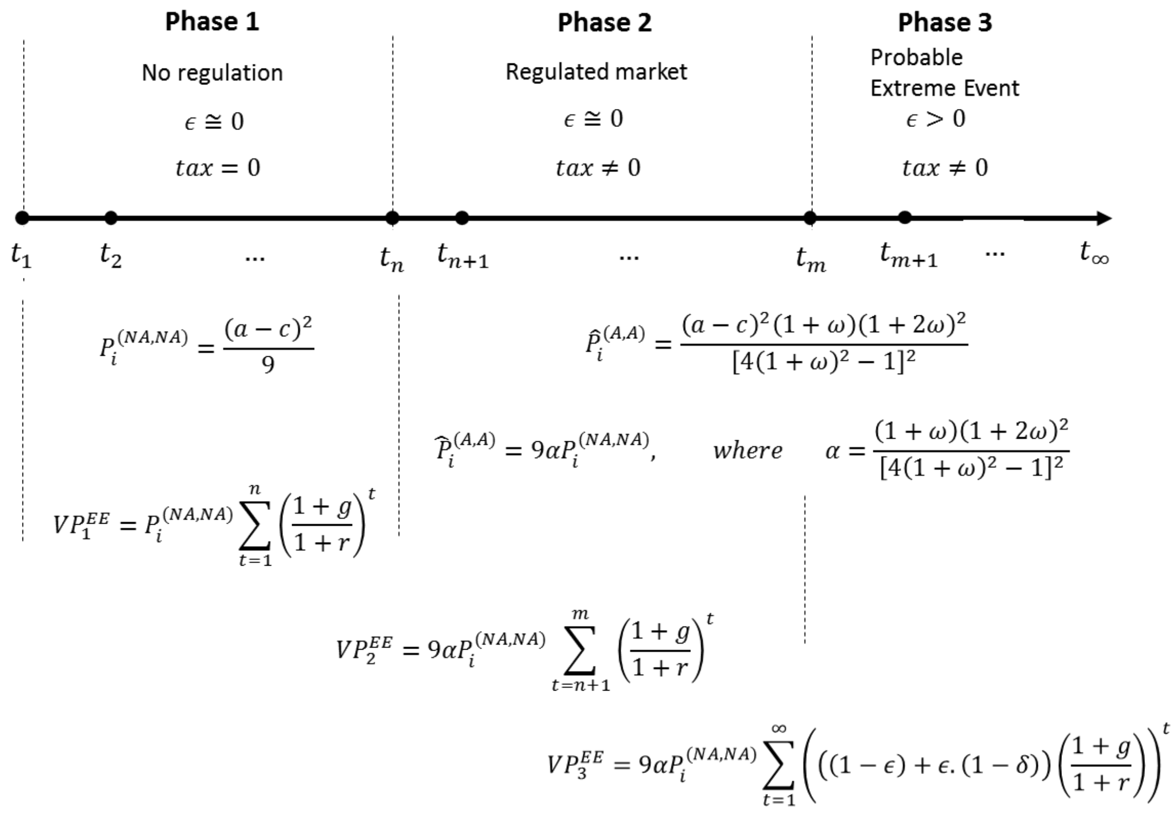

Figure 6 - Scenario EE

The first phase spreads out from t_0 up to t_n, when there is no regulation, investment in adaptation does not payoff, and firms decide not to adapt (NA, NA), as discussed before in the section 3. The payoff in the first phase is $P_i^{(NA,NA)} = \frac{(a-c)^2}{9}$, $\forall i$, as discussed in Appendix 2. Let PV_1^{EE} denote the PV under scenario EE in the first phase.

The second phase goes from t_{n+1} up to t_m, when regulation begins, investment in adaptation now payoffs, firms internalize social costs and decide to adapt (A, A). The payoff in the second phase is $\hat{P}_i^{(A,A)} = \frac{(a-c)^2(1+\omega)(1+2\omega)^2}{[4(1+\omega)^2-1]^2}$, $\forall i$, see Appendix 2 for details, as well. Let PV_2^{EE} denote the PV under scenario EE in the second phase.

Finally, the expectation of occurrence of an EE evolves, up to a moment t_m, when the probability of occurrence ϵ is significantly high. The third phase begins with the impacts of such event. From t_{m+1} on, firms start to lose cash flow by a percentage δ, due to physical or technical impairment.

Under these conditions, a depression in the value of the firms is expected. The payoff in the third phase is the same as in the second phase, however it is depressed due to the uncertainty created by the EE with probability ϵ, and the possibility of loss with percentage δ. Let PV_3^{EE} denote the PV under scenario EE in the third phase.

Let PV_{EE} denote the present value of the cash generated by the firm under the scenario EE, arising from the three phases above described. Thus, Appendix 2 details the development of the expression $PV^{EE} = PV_1^{EE} + PV_2^{EE} + PV_3^{EE}$ and the demonstration of the relationship between PV_{EE} and PV_{BAU} (Appendix 2, equation D):

$$\frac{PV^{EE}}{PV^{BAU}} = 1 + 9\alpha.\left(\frac{1+g}{1+r}\right)^n - 9\alpha.\left(\frac{1+g}{1+r}\right)^m.\frac{\epsilon.\delta(1+g)}{(r-g)+\epsilon.\delta(1+g)}$$

When $n < m$, $g < r$, $\epsilon \in (0,1)$, and $\delta \in (0,1]$.

4.2.3. Valuation under climatic distress

PV^{EE} expresses the valuation of the firm under distress due to CC and eventual EE. PV^{BAU} expresses the valuation in “on going concern” situation, or “business as usual”. In order to study the theoretical prediction, the model simulates the relationship PV_{ee}/PV_{BAU} (vertical axis), as a function of the investment in adaptation rk (horizontal axis), with the variation of five representative parameters:

- Period t_n when government is expected to begin regulation,
- Period t_m when the expectation of occurrence of EE becomes virtually certain,

- Technological efficiency e,
- Probability ε of occurrence of an EE, and
- Expectation δ of cash generation decrease,

Figure 7 shows the sensibility of PV_{ee}/PV_{BAU} according to the period t_n when starts to regulate.

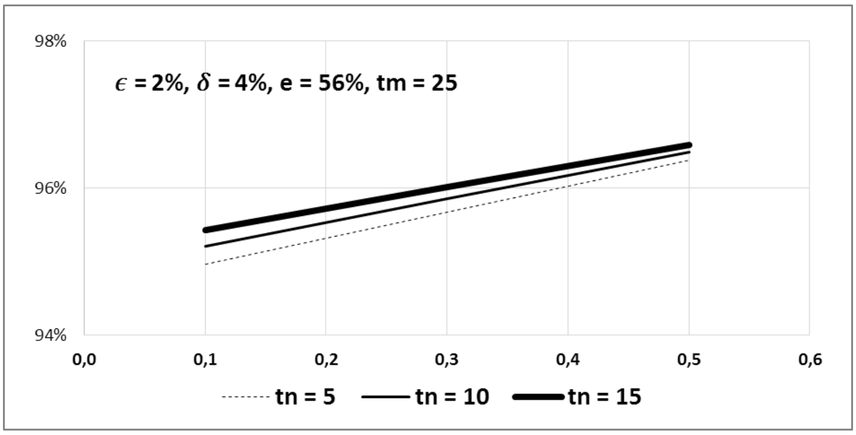

Figure 7 - PV_{ee}/PV_{BAU} sensibility to period t_n

Before regulation firms does not adapt and post better profits, which is aligned with expression P1, proposition 1. However, once regulation starts, the externality is internalized and profits will depress. The sooner the regulation begins, worse is the valuation, *ceteris paribus*.

Figure 8 shows the sensibility of PV_{ee}/PV_{BAU} according to the period t_m when the occurrence of EE is expected to significant.

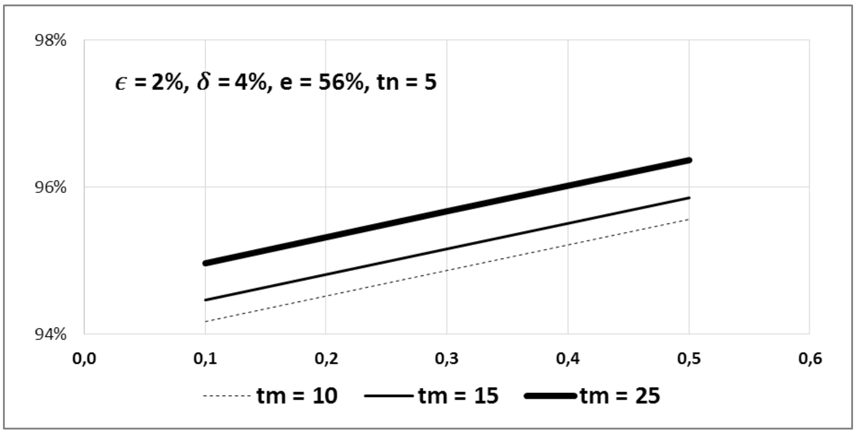

Figure 8 - PV_{ee}/PV_{BAU} sensibility to period t_m

Nature signals global warming and the science issues reports that warn on the consequences. The expectation of occurrence of EE grows bigger and bigger, years after years. In the end, the sooner the occurrence of EE is expected, worse is the valuation, *ceteris paribus*.

Figure 9 shows the sensibility of PV_{ee}/PV_{BAU} according to the technological efficiency e. If the technology is below a certain bracket, there is no point to invest in adaptation, once the investment does not payoff and the valuation decreases. Above this bracket, the bigger is the investment, the better is the valuation, *ceteris paribus*.

The most relevant aspect of this chart is to pinpoint the relevant role of the technology for the justification of the necessary investments. If the role of government is important to curb negative behaviors, it is also important to create the necessary conditions to develop appropriate solutions to face the problem.

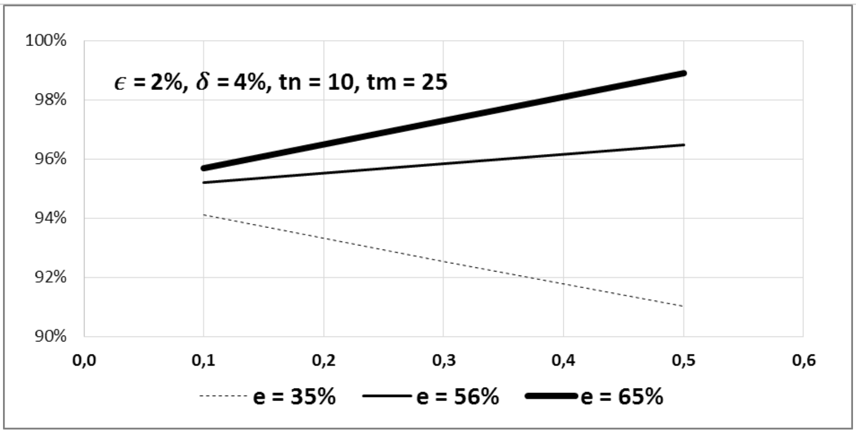

Figure 9 - PV_{ee}/PV_{BAU} sensibility to efficiency *e*

Figure 10 shows the sensibility of PV_{ee}/PV_{BAU} according to the probability ε of occurrence of an EE.

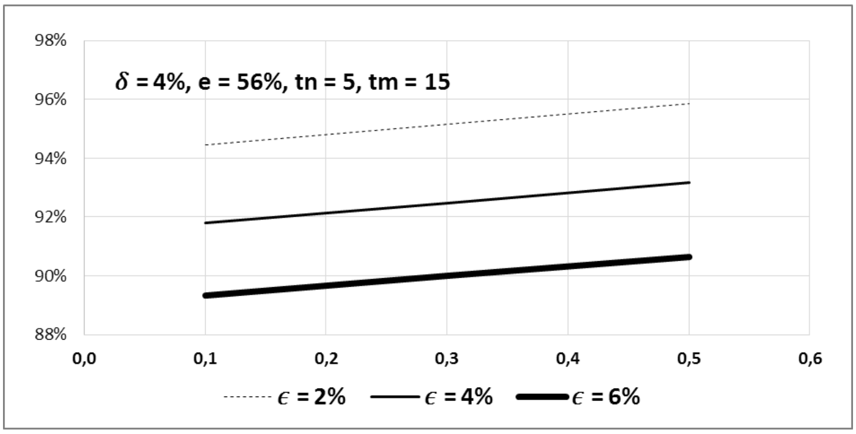

Figure 10 - PV_{ee}/PV_{BAU} sensibility to probability ε of EE

This result aligns with Figure 8, since the expectation of EE is directly related to the proximity in time of an event. The bigger is the expectation of EE, represented in the model by the probability ϵ, worse is the valuation, *ceteris paribus*.

It highlights the importance that information plays in this process. Management must be aware of the scientific warnings in order to make the best judgement in terms of expectation of EE. The sooner decision makers realize the potential distress and opportunities that surrounds the global warming, the better may the decisions and the results achieved.

In the same direction, it highlights the importance of appropriate readings from investors and market analysts. They must be prepared to anticipate the trend and properly valuate the assets exposed to climate distress.

Figure 11 shows the sensibility of PV_{ee}/PV_{BAU} according to the expectation δ of cash generation decrease.

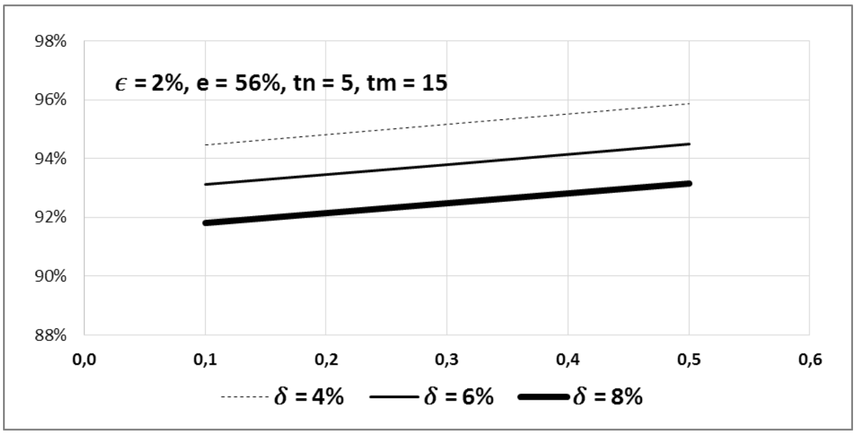

Figure 11 - PV_{ee}/PV_{BAU} sensibility to expectation δ cash generation decrease

Also aligned with Figure 8 and Figure 10, management, investors and market analyst must be able to anticipate the eventual impacts over the business operation. This is particularly important for industries exposed to the most evident threats, such as seaport, sewage and water, oil and gas companies, among many others. The bigger is the expectation of the impact, worse is the valuation.

On the other hand, all these threats also mean opportunities for new businesses that may bring alternatives to the market (see wind power and renewables), as well as new solutions coming innovation within the traditional businesses.

5. DISCUSSION

The environmental and economic systems are closed inter-connected. Pollution are produced together the outputs of firms, and are assumed to be a convex function, proportional to the square of quantities produced. With the evolution of the changes in the environmental system, “climate change,” eventually events of low probability and high impact, “extreme events,” are expected to occur. These events may impact the capability to generate cash, by firms, if they are not adapted to these new conditions. The government may regulate (or not) the market, for instance, through taxation over emissions.

5.1. FIRST OBJECTIVE – THE BEHAVIOR

The first objective is to model the strategic behavior of the firms exposed to the above-mentioned climate risks, using Cournot and game theory. With specific parameters and marginal linear payoff functions, within a non-regulated market, the model proposes that the firms would not invest in adaptation to CC, once the capital expenditures would not payoff.

When government begins to regulate the market, the model proposes that the firms would move forward to implement adaptation assets. In this regulated market, the overall quantities produced are be smaller than prior to the internalization of the social costs. In fact, when marginal cost MC shifts up by the amount of tax to $(MC + t)$, a new equilibrium establishes typically with smaller quantities. This result aligns with economic theory, presented in the literature review (ROMEIRO, 2012).

Concerning the determinants of this behavior, some factors that influence the decision to invest in adaptation may be drawn from the equation (12).

The first condition to satisfy the inequation is:

When $te < r$, then $\omega > 0, \quad \forall\, t, r, e.$

Therefore, technology and taxation are inversely proportional. When the market offers efficient solutions for adaptation, the tax bracket may lower, therefore, regulators must consider the level of technology efficiency; otherwise, the excess taxation may not produce the expected results, but destroy value creation. Therefore, without a deeper understanding about the technologies available and their efficiency, it is not easy to determine an optimal taxation level.

The second condition to satisfy (12) is:

When $0 < \frac{rk}{s} < \frac{tr}{te-r}$, then $\omega > 0, \quad \forall\, t, r, e, s, k$

The inequation shows the relationship between cost in adaption rk and the emission level slope, as a function of the tax bracket (t), the cost of capital (r), and the technology efficiency (e). Considering the tax bracket and cost of capital unchanged, as technological efficiency increases, smaller investment is needed.

Thus, the movement toward adaptation is subject to the availability of technology and its efficiency. The more efficient is the technology, smaller shall be the social costs internalized by the firms, and taxation is inversely proportional to the efficiency of technology. This result supports the theoretical discussion mentioned in the literature review (ROMEIRO, 2012), that raises capital, technology, and innovation as key factors, once they may increase the efficiency of the economy, and reduce, or even eliminate, the scarcity of natural resources.

5.2. SECOND OBJECTIVE – THE VALUATION

Another objective is to estimate firm´s valuation, when they are exposed to a probability of distress, due to climate change. The model proposes that there is a level of technological efficiency in adaptation, below which there is no point to invest in adaptation, once the valuation will decrease. Above this bracket, the valuation improves with the amount of investment. However, the valuation worsens, as early as government begins to regulate and the expectation of EE becomes significant. In addition, the valuation worsens, as much as the probability of an EE and the impact that firms are subject.

5.3. GENERALIZATION OF FINDINGS

The utilization of Cournot as the foundation for the development of the proposed model is consistent with the literature for non-repeated games, among oligopolistic firms. Moreover, this consideration does not limit the results exclusively to oligopolies (TIROLE, 1988). In fact, Cournot equilibrium generalizes for a competitive market. When the assumption of two firms is relaxed and all quantities are the same (symmetric model), the equation (8) rewrites $p = a - Q$, where $Q = \sum_{i=1}^{n} q_i = nq$.

Applying the first order maximization condition, the equation (9) rewrites $p(Q) + q\frac{\partial p}{\partial q_i} - c - \frac{\partial K_i}{\partial q_i} - \frac{\partial T_i}{\partial q_i} = 0$, $\forall\, i$. Thus $q = \left(a - c - \frac{\partial K_i}{\partial q} - \frac{\partial T_i}{\partial q}\right)/(n+1)$, $p = c + \left(a - c - \frac{\partial K_i}{\partial q} - m\frac{\partial T_i}{\partial q}\right)/(n+1)$ and $\lim_{n \to \infty} p = c$, that is, the market price tends to the competitive price c (TIROLE, 1988).

Based on this generalization, the equilibrium conditions, the determinants of firms´ behaviour, and implications over their valuation hold even for a more general condition, with many price taken firms. Therefore, the model proposed in this paper holds to the vast majority to firms in the real economy.

5.4. CONTRIBUTIONS

Sustainability is a body of knowledge typically multidisciplinary. The literature review shows the existence of several studies in economics, business management, production, marketing, natural science, among others, however an “integrated” approach, that brings together different aspects from these areas of study, is not common in the literature.

The proposed framework integrates different disciplines, namely, business and environmental economy, and build a bridge between the essential theories involved. By doing so, this framework may support researchers, asset managers, policy makers, market analysts, and managers to advance the dialogue between these disciplines, aiming to get leaders, from diverse areas, aligned with the objective of the creating new attitudes and behaviors that may support the next generations.

This research raises important discussions: the fundamental role of government and the importance of technology and innovation, to assure effectiveness of any program that aims to reduce the impact of the economic development over the quality of the environment. In summary, this model provides a comprehensive understanding about the firm´s behaviour, when facing CC, the role of the government, and potential impacts, if extreme events are expected to happen.

A second contribution is the fact the paper models the decision to adapt to CC, and its economic implications, from the investment standpoint. Typically, economy literature utilizes the pollution flow as the key variable for modelling this type of processes. Although, utilizing a different approach, the results remain consistent with previous research.

The model is not only adherent with the economic and finance theory, but strong evidences from the market support the model, as discussed ahead.

Practitioners have already begun to identify the potential risk that firms exposed to climate extremes may post, if appropriate strategic choices are postponed. The Principles for Responsible Investment (PRI) is a worldwide network of asset owners and asset managers with over 1,000 signatories representing some $35 trillion in assets. The Principles reflect a long-term view, emphasizing that environmental, social and corporate governance (ESG) issues can affect the performance of investment portfolios and require appropriate considerations by investors (LYDENBERG, 2013). Similarly, a United Nations Global Compact report highlighted that 67% of more than 1,000 CEO’s interviewed worldwide see investors integrating sustainability data in company assessment and valuation (UN Global Compact-Accenture, 2014).

Remarkably, Storebrand, a major Norwegian and Swedish pension fund with US$74 billion of assets, decided to divest from all its coal investments in July 2013 (BLOOMBERG Financial LP, 2013). Behind this movement, there is a clear preoccupation to reduce portfolio exposure to fossil fuel assets, because of concerns over stranded assets.

In September 2014, emblematic the Rockefeller Brothers Fund (RBF), a foundation created in 1940 by the sons of John D. Rockefeller Jr., introduced a divestment program from fossil fuels related investments. They are committed to reducing the exposure to coal and tar sands to less than one percent of the total portfolio by the end of 2014. Following this immediate action, RBF announced a study to determine an appropriate strategy for further divestment, over the next few years, from any remaining fossil fuel investments (RBF, 2014).

Although, this type of movement may not change dramatically the market, since the market value of RBF assets total below US$ 1 billion, it has a profound symbolic meaning.

For policy makers, the model not only reinforces the need of centralized rules to drive the economic agents, but also calls the attention for the need of incentives for the development of efficient technologies for mitigation and adaptation to climate changes. Therefore, policies must consider not only different formats to curb unfavorable behaviors, but also to encourage solutions that will make the way to go smoother.

Finally, educated consumers may play an important role in this process, as long as they can punish firms with unfavorable behavior, or encourage those who are committed to create the conditions for a long term, sustainable development.

6. CONCLUSION

Following the climate signals, the valuation of firms is now subject to new regulations, technological innovations, and change in consumer behavior (BRINKMAN et al., 2008). Despite of this trend, in general, executives, business, and management journals have so far paid little attention to the implications of the threat coming for the global warming (GOODALL, 2008). Even those who intend to use climate information to support strategic decisions, they struggle with the uncertainties (SNOVER, MANTUA, LITTELL, & ALEXANDER, 2013).

Given the long time span and the great uncertainty around the subject of climate conditions, firms face decisions that may significantly influence their value. Such decisions are strongly dependent on availability of information and management individual perceptions. Not only risks are at stake, but also opportunities to explore. Investors are more and more concerned with these developments and government has a decisive role.

The theoretical model developed is consistent with economic and management theories, however the paper avoided complex mathematical treatments, and opted for using specific parameters and linear marginal profit functions to illustrate the propositions, in a way to facilitate the communication with the intended public. This approach is also justified due to the number of parameters in the model. Any change in some of these parameters, especially the efficiency of technology e, the slope of the investment k, and the tax bracket t.

A foreseeable improvement is the introduction of a stricter approach. The mathematical development shall provide robustness to the model; however, the result may be intended to a different public.

Regarding the valuation of firms, the paper avoided real option valuation methodology. Taking into account real options may positively affect potential investments in adaptation and even change decisions, especially due the size of the adaptation assets and the significant uncertainty around the impacts of CC. In fact, the firms may either abandon the investment in adaptation, postpone or even expand the projects, provided the more favorable or unfavorable impacts do materialize in the future. Therefore, the introduction of the real option valuation is another possible evolution for this research.

Still regarding the valuation of firms, another development is to model the present value of the cash based on CVar risk measure. This measure is defined as the conditioned expectation of the loss distribution's value, worse (greater) than a given α quantile. In this context, the firms will behave based on his previous estimates of the probability function (STREET, 2010). This probability function would be defined in accordance with expected parameters related to climate, such as temperature.

The proposed model considered two oligopolistic firms, interacting in an economy administered by the government. For simplifying purposes, consumers were not included, but could have been there. In fact, the punishment (or encouragement) of the firms that do not act in accordance of the environmental demands may come from government, but also from consumers. Another evolution of this model, therefore, is to include the consumer in this theoretical economy and take in account his/her willingness to buy the firms´ products. In this scenario, the framework would encompass factors such reputation and relationship with stakeholders.

In addition, the taxation (over emissions) used in this model, to represent the necessary intervention from the government, may be discussed and deepen. Extensive literature on this subject, not used in this paper, may dialogue with the proposed model, exploring different alternatives of taxation, and discussing its pros and cons.

Under the theory of games, the game played between firms consisted of one interaction. An evolution may consider repeated games format and verify the consistency of the result.

An empirical research to validate the model is another subject for future research. Not all variables are observable and proxies may not be available, however it shall be the natural step forward.

A foreseeable approach is to compare the behavior and valuation of similar firms, from two industrialized group of countries whose the major distinction between them would be the legislation and taxation over pollution and emissions. A cross section could compare performance of similar firms, under different jurisdiction, having controlled other variables, such industry and size.

Still regarding the empirical research, another approach is to compare the behavior and valuation of similar firms before and after the introduction of major changes in legislation. West and Bianchi (2013) used this approach when studied the effects of adaptation by European Union (EU) energy companies, before and after the introduction of a CO2 new legislation in 2005, with increased significantly liabilities related to emissions. They worked with publicly listed companies split into a group of firms that “adapted” and firms that chose not to “adapt.”

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APPENDIX 1 – FIRMS' BEHAVIOR

The purpose of this Appendix 1 is to provide the mathematical foundation to support the interpretation of the behavior of the firms, prior and after the regulation of the market. The intention is to express the payoff of the firms, for every possible strategic choice $(A, NA), (NA, A)$, and (A, A), as a function of the payoff under the strategic choice (NA, NA).

Thus, the equations (10), (11) and (12) rewrite as $\max_{q_i}\left[\pi_i\left(q_i, q_j\right)\right] = \max_{q_i}\left\{\left[(a-c)-q_j\right]q_i-(1+\omega_i){q_i}^2\right\}$. Notice that ω now is noted with an underscript because it will vary according to the strategic choices $(NA, NA), (A, NA), (NA, A)$, or (A, A) of each firm. Imposing the maximization condition $\partial\pi_i\left(q_i, q_j\right)/\partial q_i = 0, \forall i$ and solving the system of equations for q_i and q_j:

$q_i^* = \frac{(a-c)(1+2\omega_i)}{\left[4(1+\omega_i)\left(1+\omega_j\right)-1\right]}\ and\ q_j^* = \frac{(a-c)\left(1+2\omega_j\right)}{\left[4(1+\omega_i)\left(1+\omega_j\right)-1\right]}$	(A)

Replacing q_i and q_j in the profit function:

$\pi_i = \frac{(a-c)^2(1+\omega_i)\left(1+2\omega_j\right)^2}{\left[4(1+\omega_i)\left(1+\omega_j\right)-1\right]^2}, \qquad \pi_j = \frac{(a-c)^2\left(1+\omega_j\right)(1+2\omega_i)^2}{\left[4(1+\omega_i)\left(1+\omega_j\right)-1\right]^2}$	(B)

1.1. GOVERNMENT DOES NOT REGULATE THE MARKET

1.1.1. Trivial case: both firms (F_1, F_2) decide not adapt (NA, NA)

$q_i^{(NA,NA)} = \frac{(a-c)}{3}, \quad \forall i$	$P_i^{(NA,NA)} = \frac{(a-c)^2}{9}, \quad \forall i$

1.1.2. F_1 decides to adapt and F_2 otherwise (A, NA) - proposition 1, corollary 1

Replacing $\omega_i = rk$ and $\omega_j = 0$ in the Equations A and B:

$q_i^{(A,NA)} = \frac{(a-c)}{[4(1+rk)-1]}$	$q_j^{(A,NA)} = \frac{(a-c)(1+2rk)}{[4(1+rk)-1]}$
$P_i^{(A,NA)} = \frac{(a-c)^2(1+rk)}{[4(1+rk)-1]^2}$	$P_j^{(A,NA)} = \frac{(a-c)^2(1+2rk)^2}{[4(1+rk)-1]^2}$

Let the cost of capital $r = 5\%$, the tax bracket $t = 10\%$, and the efficiency of the technology to adapt $e = 56\%$, in order to illustrate the results. Figure 12 depicts the expressions P1, C1 and C2 as function of the independent variable rk. Therefore, it is reasonable to propose that:

$$\frac{P_1^{(A,NA)}}{P_1^{(NA,NA)}} < 1 \; and \; \frac{P_2^{(A,NA)}}{P_2^{(NA,NA)}} > 1, \quad \{\forall rk, rk \in R^+, given \;\; r = 5\%, t = 10\%, e = 56\%\}$$

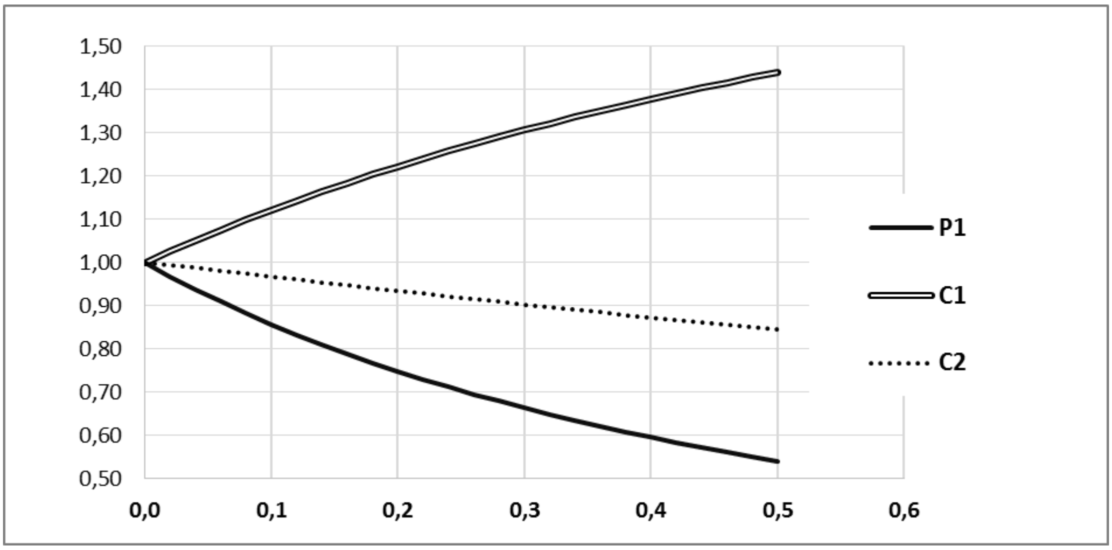

Figure 12 - Proposition 1, corollaries 1 and 2

1.1.3. Both firms (F_1, F_2) decide to adapt (A, A) – corollary 2

Replacing $\omega_i = rk$ and $\omega_j = 0$ in the Equations A and B:

$$q_i^{(A,A)} = \frac{(a-c)(1+2rk)}{[4(1+rk)^2-1]}, \qquad \forall i$$

$$P_i^{(A,A)} = \frac{(a-c)^2(1+rk)(1+2rk)^2}{[4(1+rk)^2-1]^2}, \qquad \forall i$$

Figure 5 depicts the expression C2 as function of the independent variable rk. Therefore, it is reasonable to propose for the given parameters that:

$$\frac{P_1^{(A,A)}}{P_1^{(NA,NA)}} = \frac{P_2^{(A,A)}}{P_2^{(NA,NA)}} < 1\,, \qquad \{\forall rk, rk \in R^+, given\ \ r = 5\%, t = 10\%, e = 56\%\}$$

1.2. GOVERNMENT DOES REGULATE THE MARKET

1.2.1. Trivial case: both firms (F_1, F_2) decide not adapt (NA, NA)

Replacing $k = 0$, $t > 0$ and $\omega_i = \omega_j = ts$ in the Equations A and B:

$\hat{q}_i^{(NA,NA)} = \hat{q}_j^{(NA,NA)} = \frac{(a-c)(1+2ts)}{[4(1+ts)^2-1]}$	$\hat{P}_i^{(NA,NA)} = \hat{P}_j^{(NA,NA)} = \frac{(a-c)^2(1+ts)(1+2ts)^2}{[4(1+ts)^2-1]^2}$

1.2.2. F_1 decides to adapt and F_2 otherwise (A, NA) – proposition 2, corollary 3

Replacing $\omega_i = \omega = rk + t(s - ek)$ and $\omega_j = ts$ in the Equations A and B:

$\hat{q}_i^{(A,NA)} = \frac{(a-c)(1+2ts)}{[4(1+\omega)(1+ts)-1]}$	$\hat{q}_j^{(A,NA)} = \frac{(a-c)(1+2\omega)}{[4(1+\omega)(1+ts)-1]}$
$\hat{P}_i^{(A,NA)} = \frac{(a-c)^2(1+\omega)(1+2ts)^2}{[4(1+\omega)(1+ts)-1]^2}$	$\hat{P}_j^{(A,NA)} = \frac{(a-c)^2(1+ts)(1+2\omega)^2}{[4(1+\omega)(1+ts)-1]^2}$

Assume same parameters $r = 5\%, t = 10\%$, and $e = 56\%$, in order to illustrate the results, in order to illustrate the results. Figure 13 depicts the expressions P2 and C3 as function of the independent variable rk. Therefore, it is reasonable to propose for the given parameters that:

$$\frac{\hat{P}_i^{(A,NA)}}{\hat{P}_i^{(NA,NA)}} < 1 \; and \; \frac{\hat{P}_j^{(A,NA)}}{\hat{P}_j^{(NA,NA)}} > 1 , \qquad \{\forall rk, rk \in R^+, given \;\; r = 5\%, t = 10\%, e = 56\%\}$$

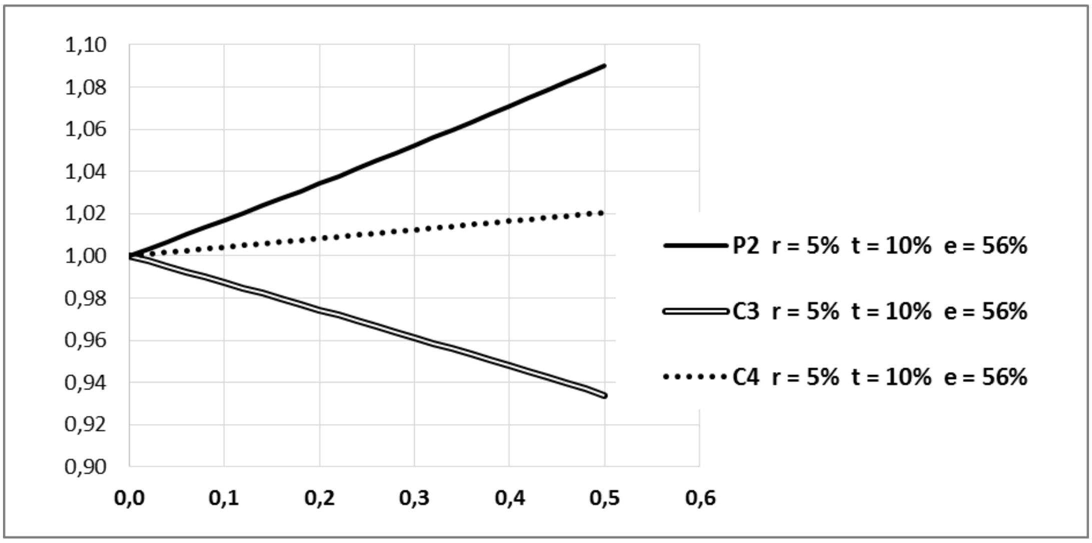

Figure 13 - Proposition 2, corollaries 3 and 4

1.2.3. Both firms ($\boldsymbol{F_1, F_2}$) decide to adapt ($\boldsymbol{A, A}$) – corollary 4

Replacing $\omega_{ij} = \omega = rk + t(s - ek)$ in the Equations A and B:

$\hat{q}_i^{(A,A)} = \frac{(a-c)(1+2\omega)}{[4(1+\omega)^2 - 1]}, \forall i$	$\hat{P}_i^{(A,A)} = \frac{(a-c)^2(1+\omega)(1+2\omega)^2}{[4(1+\omega)^2 - 1]^2}, \forall i$	(D)

Assume same parameters $r = 5\%, t = 10\%$, and $e = 56\%$, in order to illustrate the results. Figure 13 also depicts the expression C4 as function of the independent variable rk. Therefore, it is reasonable to propose for the given parameters that:

$$\frac{\hat{P}_1^{(A,A)}}{\hat{P}_1^{(NA,NA)}} = \frac{\hat{P}_2^{(A,A)}}{\hat{P}_2^{(NA,NA)}} > 1\,, \qquad \{\forall rk, rk \in R^+, given\; r = 5\%, t = 10\%, e = 56\%\}$$

1.3. QUANTITIES OF EQUILIBRIUM

Building on the expressions previously discussed, and assuming same parameters as before, Figure 14 depicts the expression C5 as function of the independent variable rk. Therefore, it is reasonable to propose for the given parameters that:

$$\frac{\hat{q}_1^{(A,A)} + \hat{q}_2^{(A,A)}}{q_1^{(NA,NA)} + q_2^{(NA,NA)}} < 1\,, \qquad \{\forall rk, rk \in R^+, given\, r = 5\%, t = 10\%, e = 56\%\}$$

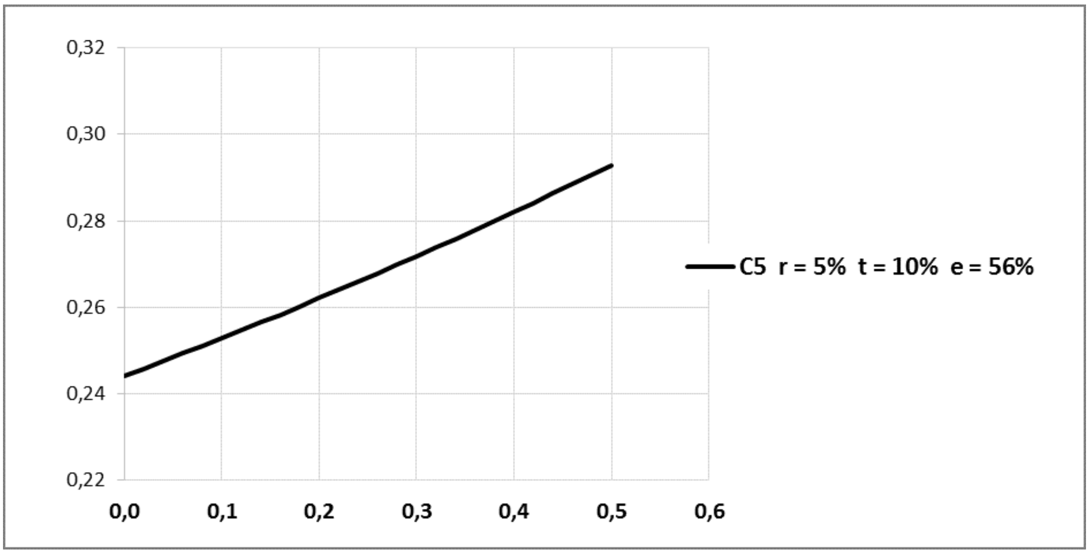

Figure 14 – Corollary 5

APPENDIX 2 – FIRMS' VALUATION

The purpose of this Appendix 2 is to launch the mathematical foundation to support the valuation of the firms under CC related distress. The intention is to describe the PV of the distressed scenario (EE) as a function of the PV under on going concern scenario (BAU).

2.1. PRESENT VALUE UNDER SCENARIO BAU

In this section, I model the PV for the scenario BAU, based on the traditional DCF formulation (see Figure 5).

Without loss of generality, the cash flow at the beginning of the valuation period t_0 equals and the cash flow forecasted for t_1 is $\left(\frac{1+g}{1+r}\right)$, where g is the expected growth fixed rate and r is the discount rate. Thus, the trivial expression of the discount factor for an infinite cash flow is $\sum_{t=1}^{\infty}\left(\frac{1+g}{1+r}\right)^{t} = \frac{(1+g)}{(r-g)}$.

With Figure 5 in mind, the cash flow at the beginning of the valuation period t_0 equals $P_i^{(NA,NA)} = \frac{(a-c)^2}{9}$ (Appendix 1, equation C). Let PV^{BAU} denote the PV under scenario BAU, then $PV^{BAU} = P_i^{(NA,NA)}.\frac{(1+g)}{(r-g)}$.

2.2. PRESENT VALUE UNDER SCENARIO EE

In this section, the intention is to find $PV^{EE} = PV_1^{EE} + PV_2^{EE} + PV_3^{EE}$, where PV_1^{EE} is the PV of the first phase, PV_2^{EE} the PV of the second phase, and so on. Refer to Figure 6.

Saha and Makiel (2012) and Damodaran (2002, 2006) are the basis to model the PV for the scenario EE. Consider that, at each period, there is a probability ϵ that the cash flow will cease to exist, then discount factor is $\sum_{t=1}^{\infty}(1-\epsilon)^t.\left(\frac{1+g}{1+r}\right)^t = \frac{(1-\epsilon)(1+g)}{(r-g+\epsilon+\epsilon.g)}$. In addition, consider that, at each period, there is a probability ϵ of cash flow being reduced by δ, that is, at any period the cash flow has a probability of realizing a value of $(1-\delta)$. Under this framework, the discount factor is $\sum_{t=1}^{\infty}\left(\left((1-\epsilon)+\epsilon.(1-\delta)\right)\left(\frac{1+g}{1+r}\right)\right)^t$. The trivial DCF holds when $\epsilon = 0$ or $\delta = 0$.

With Figure 6 in mind, there are three phases:

- Phase one (t_1 up to t_n): the cash flow is the same in scenario BAU. In t_0, the cash flow equals $P_i^{(NA,NA)} = \frac{(a-c)^2}{9}$ (appendix 1, equation C) and in t_1, $P_i^{(NA,NA)}.\left(\frac{1+g}{1+r}\right)$. Using a geometric series with a constant ratio between successive terms, the present value from periods t_1 up to t_n is $PV_1^{EE} = P_i^{(NA,NA)}.\left(\frac{1+g}{r-g}\right).\left[1-\left(\frac{1+g}{1+r}\right)^n\right] = PV^{BAU}.\left[1-\left(\frac{1+g}{1+r}\right)^n\right]$.

- Phase two (t_{n+1} up to t_m): the cash flow is smaller than in scenario BAU and in t_n equals $\hat{P}_i^{(A,A)} = \frac{(a-c)^2(1+\omega)(1+2\omega)^2}{[4(1+\omega)^2-1]^2}$ (appendix 1, equation D). This expression may be rewritten as $\hat{P}_i^{(A,A)} = \alpha.(a-c)^2 = 9\alpha.P_i^{(NA,NA)}$, where $\alpha = \frac{(1+\omega)(1+2\omega)^2}{[4(1+\omega)^2-1]^2}$. In t_{n+1}, cash flow equals $9\alpha.P_i^{(NA,NA)}.\left(\frac{1+g}{1+r}\right)$. Using a geometric series with a constant ratio between successive terms, the PV from t_{n+1} up to t_m is $PV_2^{EE} = 9\alpha.P_i^{(NA,NA)}.\left(\frac{1+g}{r-g}\right).\left[\left(\frac{1+g}{1+r}\right)^n - \left(\frac{1+g}{1+r}\right)^m\right] = 9\alpha.PV^{BAU}.\left[\left(\frac{1+g}{1+r}\right)^n - \left(\frac{1+g}{1+r}\right)^m\right]$.

- Phase three (t_{m+1} up to t_∞): the cash flow is the same as phase two, however now there is a probability ϵ of cash flow being reduced by δ. Using the same method as before, the present value from t_{m+1} up to t_∞ is $PV_3^{EE} = 9\alpha.P_i^{(NA,NA)}.\left(\frac{1+g}{1+r}\right)^{m+1}.\frac{(r-g)}{(r-g)-\epsilon.\delta(1+g)} = 9\alpha.PV^{BAU}.\left(\frac{1+g}{1+r}\right)^m.\frac{(r-g)}{(r-g)+\epsilon.\delta(1+g)}$.

  Finally, adding up the three phases of scenario EE:

$$PV^{EE} = PV_1^{EE} + PV_2^{EE} + PV_3^{EE}$$

$$PV^{EE} = PV^{BAU}.\left[1 - \left(\frac{1+g}{1+r}\right)^n\right] + 9\alpha.PV^{BAU}.\left[\left(\frac{1+g}{1+r}\right)^n - \left(\frac{1+g}{1+r}\right)^m\right] + 9\alpha.PV^{BAU}.\left(\frac{1+g}{1+r}\right)^m.\frac{(r-g)}{(r-g)+\epsilon.\delta(1+g)}$$

  Where $\alpha = \frac{(1+\omega)(1+2\omega)^2}{[4(1+\omega)^2-1]^2}$.

2.3. RELATIONSHIP BETWEEN $\boldsymbol{PV^{EE}}$ AND $\boldsymbol{PV^{BAU}}$

Dividing all terms by PV^{BAU}:

$$\frac{PV^{EE}}{PV^{BAU}} = \left[1 - \left(\frac{1+g}{1+r}\right)^n\right] + 9\alpha.\left[\left(\frac{1+g}{1+r}\right)^n - \left(\frac{1+g}{1+r}\right)^m\right] + 9\alpha.\left(\frac{1+g}{1+r}\right)^m.\frac{(r-g)}{(r-g)+\epsilon.\delta(1+g)}$$

$$\frac{PV^{EE}}{PV^{BAU}} = 1 + (9\alpha - 1)\left[\left(\frac{1+g}{1+r}\right)^n\right] + 9\alpha.\left(\frac{1+g}{1+r}\right)^m\left[\frac{(r-g)}{(r-g)+\epsilon.\delta(1+g)} - 1\right]$$

Finally, the relationship between PV^{EE} and PV^{BAU}:

$\frac{PV^{EE}}{PV^{BAU}} = 1 + 9\alpha.\left(\frac{1+g}{1+r}\right)^n - 9\alpha.\left(\frac{1+g}{1+r}\right)^m.\frac{\epsilon.\delta(1+g)}{(r-g)+\epsilon.\delta(1+g)}$	(D)