Determinacy and learnability of equilibrium in a small-open economy with sticky wages and prices

Eurilton Araújo *

Central Bank of Brazil, Brasília, Brazil FUCAPE Business School, Vitória, Brazil

A R T I C L E   I N F O

Article history:
Accepted 27 July 2015
Received in revised form 27 April 2016
Received 27 July 2015
Available online 5 May 2016

JEL Classification:
E13
E31
E52
F41

Keywords:
Determinacy
Inflation
Learnability
Monetary policy rules

A B S T R A C T

In a small-open economy model with nominal wage and price rigidities, it has been argued that, in terms of welfare losses, the monetary policy rule that responds to consumer price index (CPI) inflation performs better than rules that react to competing inflation measures. From the viewpoint of determinacy and learnability of rational expectations equilibrium (REE), this paper suggests that the rule that responds to CPI inflation does not increase the Central Bank's ability to promote the convergence of an economy to a determinate and learnable REE nor improves the speed of this convergence when compared with rules that react to contending inflation measures.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Despite the behavior of many inflation-targeting central banks, most research on small-open economies with sticky prices, such as Gal and Monacelli (2005), suggests that the monetary authority should respond to domestic inflation rather than to consumer price index (CPI) inflation. In a small open economy with sticky wages and prices, Rhee and Turdaliev (2013) and Campolmi (2014) compared monetary policy rules that reacted to different inflation measures according to their welfare losses. In this context, they found that CPI inflation performed better than some contending measures, including domestic inflation.

Sticky nominal wages alter the dynamics of the Gal and Monacelli (2005) model. In the presence of nominal wage rigidity, fluctuations in CPI inflation induce movements in real wages, which affect wage markups. Hence, changes in wage markups lead to fluctuations in wage inflation and in firms' marginal costs. Therefore, in contrast to Gal and Monacelli (2005), there is a direct effect of CPI inflation on domestic price inflation.

In addition, since CPI inflation depends on movements in the real exchange rate or in the terms of trade, foreign shocks, by changing these variables, immediately influence domestic inflation dynamics. This mechanism is the direct exchange rate channel to domestic price inflation. This channel emerges through staggered wage contracting in small open economies and it is absent in standard new Keynesian open-economy models without sticky nominal wages.

* Central Bank of Brazil, Research Department, Setor Bancário Sul (SBS), Quadra 3, Bloco B, Edifício-Sede, Brasília DF 70074-900, Brazil.
E-mail address: eurilton@gmail.com.

http://dx.doi.org/10.1016/j.iref.2016.04.014
1059-0560/© 2016 Elsevier Inc. All rights reserved.
As pointed out by Campolmi (2014), in response to domestic and foreign shocks, movements in CPI inflation translate into more volatile wage and domestic inflation rates. In this context, the central bank improves welfare by reacting to CPI inflation since this reaction reduces the volatility of wage inflation and the volatility of domestic inflation. By decreasing these volatilities, the central bank promotes reductions in wage and price dispersions, which enhance welfare.

In addition to its repercussion on welfare, the change in model dynamics engendered by the introduction of sticky nominal wages has potential implications for determinacy and E-stability of equilibrium (REE), the following question: which inflation index should monetary policy rules react to?

In contrast to Rhee and Turdaliev (2013) and Campolmi (2014), I emphasize determinacy and learnability properties as performance criteria to address the question of which inflation measure should a central bank respond to. Indeed, I present numerical results on determinacy and E-stability of equilibria for a small open economy model with sticky nominal wages and prices in which interest rate rules respond to alternative inflation measures.

This paper also addresses the effects of introducing nominal wage rigidity and trade openness on determinacy and learnability conditions associated with competing interest rate rules. Moreover, following Ferrero (2007) and Christev and Slobodyan (2014), I investigate how the specification of interest rate rules matters for the speed of convergence of an economy to a determinate and E-stable REE through an adaptive learning process. In fact, the speed of learning is an additional yardstick through which I evaluate monetary policy rules.

The main finding of this paper suggests that, when compared with rules that react to competing inflation measures, the rule that responds to CPI inflation does not provide any noticeable improvement in the central bank’s ability to promote convergence of an economy to a determinate and learnable REE. Moreover, for this rule, the learning algorithm converges slowly or with the same speed implied by alternative rules. Hence, in contrast to the evaluation based on welfare losses, the rule that responds to CPI inflation does not exhibit superior performance from the viewpoint of determinacy and E-stability.

This study is closely related to Llosa and Tuesta (2008) and Best (2015). Llosa and Tuesta (2008) studied the small open economy model developed by Gal and Monacelli (2005) and showed that the degree of openness interacted with particular interest rate rules, which may respond to exchange rates, to change the relevance of the Taylor principle as a condition ensuring determinacy and E-stability of REE. In a previous paper, Linnemann and Schabert (2006) found similar results by studying a more restricted class of interest rate rules. Best (2015) studied a closed economy model with nominal wage and price rigidities as described in Erceg et al. (2000). The paper proceeds as follows. Section 2 sets out the model. Section 3 presents the numerical findings on determinacy and E-stability of REE. Section 4 discusses the speed of convergence of determinate and E-stable equilibria. Finally, the last section concludes.

2. The model

In this section, I present the log-linear approximation of the model investigated in Rhee and Turdaliev (2013) and Campolmi (2014)\(^2\). This model is the small-open economy studied in Gal and Monacelli (2005) with the labor market characteristics found in Erceg et al. (2000), i.e., there is monopolistic competition in the labor market and households set nominal wages following the scheme proposed by Calvo (1983)\(^3\). I also discuss alternative interest rate rules representing how the central bank conducts monetary policy and I then report how I calibrate the parameters.

2.1. The private sector equilibrium conditions

After the log-linearization around the steady state of the equilibrium conditions exposed in Appendix A, the following equations represent the small open economy:

\[
\begin{align*}
\dot{y}_t & = E_t (\tilde{y}_{t+1}) - \frac{1}{\sigma_\alpha} \left[ r_t - E_t (\pi_{H_{t+1}}) \right] + \frac{1}{\sigma_\alpha} \rho r_t^n \\
\pi_{H_t} & = \beta E_t (\pi_{H_{t+1}}) + \kappa_{ph} \tilde{y}_t + \lambda_{ph} \tilde{w}_t
\end{align*}
\]

\(^1\) Flaschel, Franke, and Proaño (2008) studied a similar model in continuous time.

\(^2\) Llosa and Tuesta (2008) and Best (2015) are special cases of this model. In fact, the model in Llosa and Tuesta (2008) corresponds to setting the degree of wage rigidity to zero whereas the specification in Best (2015) is equivalent to the situation of zero degree of openness.

\(^3\) Regarding the introduction of nominal wage rigidity, Adolfson, Lassén, Lindé, and Villani (2008); Jääskelä and Nimark (2011), and Dong (2013) pointed out that this type of nominal rigidity improved the ability of open-economy medium-scale models to fit the data.
\[ \pi_{W,t} = \beta E_t(\pi_{W,t+1}) + \kappa_w \hat{y}_t - \lambda_w W_t \]  

(3)

\[ \tilde{w}_t = \tilde{w}_{t-1} + \bar{\pi}_{W,t} - \pi_{\text{CPI},t} - \Delta \pi_{n,t} \]  

(4)

\[ \pi_{\text{CPI},t} = \pi_{H,t} + \alpha (s_t - s_{t-1}) \]  

(5)

\[ \hat{y}_t = \frac{1}{\alpha} s_t + \gamma \hat{y}_t - \gamma n_t \]  

(6)

The endogenous variables \( \hat{y}_t, \pi_{H,t}, \pi_{W,t}, \tilde{w}_t, r_t, \pi_{\text{CPI},t}, \) and \( s_t \) stand for the domestic output gap, domestic inflation, nominal wage inflation, the real wage gap, the domestic interest rate, CPI inflation, and the terms of trade. The exogenous variables \( r^*_t, \Delta \pi^*_t, \gamma^*_t, \) and \( y^*_t \) denote the natural level of the real interest rate, the variation in the natural real wage, the natural level of output and foreign output.\(^4\) I assume that mutually independent first-order autoregressive processes characterize the dynamics of \( r^*_t, \Delta \pi^*_t, \gamma^*_t, \) and \( y^*_t, \) while keeping \( y^*_t \) constant at its steady state level.

The Euler equation is expression (1), which summarizes households’ intertemporal consumption decisions. Eq. (2) defines the new Keynesian Phillips curve and sums up the price-setting behavior of monopolistic firms. Eq. (3) characterizes the evolution of the wage inflation and Eq. (4) describes real wage dynamics. These expressions are implications of households’ optimal wage setting. Finally, Eq. (5) defines CPI inflation and Eq. (6) results from the assumption of complete markets and market clearing conditions in the domestic economy.\(^5\) The expectation operator \( E_t \) represents rational expectations as well as alternative expectation formation mechanisms.

The inspection of Eqs. (2)–(5) reveals how the direct exchange rate channel operates. In Eq. (4), movements in CPI inflation affect the real wage gap, which directly influences domestic price inflation and wage inflation through Eqs. (2) and (3). Eq. (5) shows that endogenous movements in the terms of trade can trigger variations in CPI inflation with an immediate effect on domestic price inflation.\(^6\)

The parameter \( \beta \) stands for the discount factor, \( \sigma \) measures the degree of relative risk aversion, \( \psi \) is the inverse of the labor supply elasticity, \( \gamma \) is the elasticity of substitution between imported goods, \( \eta \) is the elasticity of substitution between domestic and foreign goods, and \( \alpha \) measures the degree of trade openness. The degrees of price and wage stickiness are \( \theta_p \) and \( \theta_w \). Finally, the parameter \( \epsilon_w \) stands for the elasticity of substitution between distinct types of labor in the monopolistic competitive labor market.

The symbols \( \lambda_{ph}, \lambda_w, \Omega, \alpha_{\lambda}, \sigma_{\lambda}, \kappa_{ph}, \kappa_w \) denote convolutions of the deep parameters. The expressions defining them are:

\[ \lambda_{ph} = \frac{(1 - \theta_{ph})(1 - \beta \theta_{ph})}{\theta_{ph}} \lambda_w = \frac{(1 - \theta_{ph})(1 - \beta \theta_{ph})}{\theta_{ph}(1 + \epsilon_w \psi)} \Omega = \gamma \sigma + (1 - \alpha)(\eta \sigma - 1), \]

\[ \sigma_{\lambda} = \frac{\sigma}{(1 - \alpha) + \alpha \Omega}, \kappa_{ph} = \alpha \sigma_{\lambda} \lambda_{ph} \text{ and } \kappa_w = (\sigma - \alpha \sigma_{\lambda} \Omega + \psi) \lambda_w. \]

2.2. Monetary policy

I assume that the central bank follows an interest rate rule. As in Llosa and Tuesta (2008), I consider two alternative specifications for interest rate rules: contemporaneous data and forecast-based data.

The expression for the contemporaneous specification is

\[ r_t = \phi_r r_{t-1} + \phi_p \pi_{m,t} + \phi_y \hat{y}_t \]  

(7)

The forecast-based data interest rate rule is

\[ r_t = \phi_r r_{t-1} + \phi_p E_t \pi_{m,t+1} + \phi_y E_t \hat{y}_{t+1} \]  

(8)

The variable \( \pi_{m,t} \) stands for some particular inflation measure \( m \). I consider the following inflation measures: domestic inflation \( (\pi_{H,t}) \), CPI inflation \( (\pi_{\text{CPI},t}) \), and nominal wage inflation \( (\pi_{W,t}) \).\(^7\) The parameters describing the rules are \( \phi_r \), capturing interest rate inertia, and the coefficients \( \phi_p \) and \( \phi_y \) reflecting the response of the interest rate to the variables \( \pi_{m,t} \) and \( \hat{y}_t \), or, in forecast-based rules, to their expected future values. For each rule above, I consider two cases: the benchmark situation \( (\phi_r = 0) \) and the interest rate smoothing specification \( (\phi_r = 0.65) \).

---

\(^4\) The natural level of an economic variable is its equilibrium value in the absence of nominal rigidities. The gap is the difference between a given variable and its natural level.

\(^5\) The correspondence between expressions (1)–(6) and the equations in Rhee and Turdaliev (2013) goes as follows. Expression (6) is equation (16) on page 312. Expressions (2) and (3) are equations (22) and (23) on page 313. Finally, expressions (1), (4) and (5) are on page 314.

\(^6\) The expression \( q_s = (1 - \alpha) s_t \) relates the real exchange rate \( (q_s) \) to the terms of trade \( (s_t) \). Hence, these variables move in tandem.

\(^7\) Rhee and Turdaliev (2013) and Campolmi (2014) looked at these three inflation indices.
Eqs. (1)–(6) and a given interest rate rule comprise the dynamic system of equations that summarizes the artificial economy in Rhee and Turdaliev (2013). To study determinacy and learnability of REE, I parsimoniously represent this system by substituting out some endogenous variables. Appendix B discusses the system reduction and shows the matrices characterizing the parsimonious representation.

2.3. Calibration

In the baseline calibration of the model, I choose the parameters as follows.

- **Preferences.** Following Rhee and Turdaliev (2013) and Campolmi (2014), I set the elasticity of labor supply to 3 by specifying $\varphi = 3$, and the discount factor is $\beta = 0.99$. The coefficient of risk aversion is $\sigma = 5$, which is the value employed by Llosa and Tuesta (2008).
- **Goods and labor markets.** Again, I stick to the values found in Rhee and Turdaliev (2013) and Campolmi (2014). Therefore, $\delta_w = 6$ and $\theta_m = \theta_w = 0.75$.
- **Open-economy parameters.** Following Gal and Monacelli (2005); Llosa and Tuesta (2008) and Campolmi (2014), I set $\sigma = 0.4$. Agreeing with Llosa and Tuesta (2008), I set $\gamma = 1$ and $\eta = 1.5$.

3. Determinacy and E-stability

Equilibrium determinacy and learnability, also known as expectational stability (E-stability), have become important criteria for the design and evaluation of monetary policy rules in new Keynesian models. By definition, a determinate REE is unique, free from self-fulfilling fluctuations, and non-explosive. In addition, a REE is E-stable if agents who do not initially possess rational expectations coordinate upon it after using least-squares adaptive learning methods to acquire knowledge about the law of motion governing macroeconomic dynamics.

Evans and Honkapohja (2001) and Bullard and Mitra (2002, 2007) provided the foundations to analyze determinacy and E-stability of equilibria under the assumption of rational expectations. Since the publication of these papers, a burgeoning literature examining conditions that ensure determinacy and E-stability of REE in extensions of the new Keynesian model has emerged.8 Next, I summarize the conditions for determinacy and learnability in Evans and Honkapohja (2001).

3.1. Methodology

I consider the following system of linear stochastic difference equations:

$$
\begin{align*}
    x_t &= BE_t x_{t+1} + Dx_{t-1} + Kv_t \\
    v_t &= Rv_{t-1} + \zeta_t,
\end{align*}
$$

where $x_t$ is a $m \times 1$ vector of endogenous variables and $v_t$ is a $k \times 1$ vector of exogenous disturbances.

For determinacy analysis, I write the system above in the following compact form:

$$
E_t z_{t+1} = J_1 z_t + J_2 v_t
$$

where $J_1$ and $J_2$ are functions of the matrices defining the original system and $z_t = [x_t x_{t-1}]'$.

According to Farmer (1999), the REE is determinate if the number of stable eigenvalues of $J_1$ is equal to the number of predetermined variables in $z_t$.

The method for E-stability analysis follows the standard approach of Evans and Honkapohja (2001), which I summarize below. Under adaptive learning, economic agents use recursive least-squares updating to form expectations. They have a forecasting model known as the perceived law of motion (PLM), which is based on the MSV (minimum state variable) solution of the linear system of rational expectations.

The MSV solution has the form: $x_t = a + bx_{t-1} + cv_t$, where $a$ is $m \times 1$, $b$ is $m \times m$, and $c$ is $m \times k$.

The assumptions about agents’ information set at time $t$ are important in deriving E-stability conditions. In this paper, the information set is the same used in Llosa and Tuesta (2008) and Best (2015) and corresponds to the vector $(1, x_{t-1}', v_t')$. Under this time $t$ information set, the expectations are $E_t x_{t+1} = (I + b)a + b^2 x_{t-1} + (bc + cR)v_t$, where $I$ denotes the identity matrix.

The insertion of the expectations $E_t x_{t+1}$ above in the original system gives the actual law of motion (ALM): $x_t = B(I + b)a + (Bb^2 + D)x_{t-1} + (Bbc + BcR + K)v_t$.

---

8 Just to cite a few papers: Duffy and Xiao (2011) investigated the effects of capital accumulation, Kurozumi and Van Zandweghe (2012) and Best (2015) studied the role of labor market frictions. Finally, Linnemann and Schabert (2006), Llosa and Tuesta (2008), and Bullard and Schaling (2009) exemplified the research on determinacy and learnability of equilibria in open-economy models.
The perceived law of motion (PLM) used in the least-squares learning algorithm is the MSV solution and the map from PLM to ALM is

\[ T(a, b, c) = (B(l + b)a, Bb + D, Bc + BcR + K) \]

The vector \((\pi, \delta, \sigma)\) is the fixed point of the map from PLM to ALM, known as the T-map, and corresponds to a REE of the original system.

Evans and Honkapohja (2001) stated the conditions for E-stability, governing the convergence of the least-squares learning algorithm to a REE. To check these conditions, I have to compute the following derivative matrices:\(^9\) \(DT_a(\tilde{\alpha}, \tilde{\delta}) = B(l + \tilde{b}), DT_b(\tilde{\delta}) = Bb + B\delta\) + \(D\), and \(DT_c(\tilde{x}, \tilde{\sigma}) = K\) + \(D\delta\), where the operator \(\otimes\) denotes the Kronecker product. Finally, the REE solution of the original system is E-stable or learnable under the following conditions: all real parts of the eigenvalues of the derivative matrices above are lower than 1.

3.2. Results

The derivation of analytical results is not always possible, except in some special cases, such as the small open economy model investigated in Llosa and Tuesta (2008). Indeed, the addition of sticky nominal wages increases the dimension of the system and poses great challenges for dimension reduction, which would facilitate analytical results. For this reason, I adopt a simulation approach and provide numerical findings.\(^10\)

Figs. 1–6 show the regions of determinacy and E-stability associated with alternative specifications for monetary policy rules described by Eqs. (7) and (8).\(^11\) Under the baseline calibration, I investigate a benchmark rule in which I fix \(\phi_1 = 0\) and a rule with interest rate smoothing (IRS) in which I set \(\phi_2 = 0.65\), following the specification in Best (2015).

Moreover, I also study how the combination of staggered wage contracts and the degree of trade openness affects determinacy and learnability areas. In fact, these features are responsible for the emergence of the direct exchange rate channel discussed in the introduction.

3.2.1. Contemporaneous data rules

Fig. 1 illustrates the regions of determinacy and E-stability for the contemporaneous specification of the interest rate rule under the baseline calibration discussed in subsection 2.3. Figs. 2 and 3 point out the effects of different degrees of trade openness and nominal wage rigidity on the areas related to determinate and E-stable equilibria under the benchmark contemporaneous rule.

Figs. 1–3 show that the regions of determinacy and E-stability are the same under different inflation measures. This feature indicates that what is relevant for determinacy and learnability is the central bank’s ability to respond strongly to movements in some nominal anchor. Hence, responding to CPI inflation does not improve the central bank’s ability to promote the convergence of an economy to a determinate and E-stable REE.

Fig. 1 indicates the enlargement of the regions of determinacy and E-stability if the monetary policy rule features IRS. In addition, Figs. 2 and 3 show that a high degree of trade openness and an increase in nominal wage rigidity enlarge the regions of determinacy and E-stability. Furthermore, in the simulations, determinate equilibria are always E-stable and indeterminate equilibria are always E-unstable, irrespective of the monetary policy rule considered.

Some findings in Figs. 1–3 are already known in the literature. Indeed, the right column of Fig. 2 considers the benchmark rule in a closed-economy model studied by Best (2015) and the right column of Fig. 3 focuses on the benchmark rule in an small-open economy without nominal wage rigidity studied by Llosa and Tuesta (2008). In both cases, Figs. 2 and 3 endorse the findings of these previous papers.

To understand the role of central banks in preventing self-fulfilling prophecies, consider a sudden surge of inflationary expectations with no relations to any fundamental disturbances. Figs. 1–3 show that the central bank’s response has to be sufficiently aggressive to generate high real interest rates to reduce aggregate demand and bring inflation down. This aggressive response imposes restrictions on the parameters of the interest rate rule in order to guarantee determinacy and E-stability. Indeed, under this aggressive response, the rise in inflation expectations starts a sequence of events that leads to a fall in actual inflation due to a rise in the real interest rate that slows spending, causing a reduction in the output gap. After seeing this fall in inflation, people’s higher inflation expectations would cease before they become a source of macroeconomic instability.

Next, I summarize the events leading to a fall in actual inflation after the central bank aggressively increases the interest rate in response to an extrinsic rise in inflation expectations. After the rise in the interest rate, there is a positive interest rate differential with respect to the rest of the world. This differential leads now to a currency appreciation, followed by expectations of a future depreciation. The actual appreciation, by changing the terms of trade, induces an expenditure-switching effect from domestic toward foreign goods. This effect slows domestic demand and reduces the output gap. The decrease in the output gap implies less pressure on marginal costs and promotes a fall in domestic inflation. The currency appreciation and the drop in domestic inflation reduce CPI inflation. Finally, the presence of nominal wage rigidity and the reduction in CPI increase real

---

\(^9\) These derivative matrices correspond to the Jacobian of the T-map.

\(^10\) This same approach is present in some other papers that have investigated complex models, such as Duffy and Xiao (2011) and Kurozumi and Van Zandwaghe (2012). In particular, I consider a grid for the pair \((\phi_1, \phi_2)\), comprising 250,000 parameter configurations.

\(^11\) The figures are displayed at the end of the paper.
wages. Higher real wages imply higher wage markups, which are distant from the desired constant level. To adjust wage markups, workers choose to demand lower nominal wages, reducing current wage inflation.

If the central bank is sufficiently aggressive, all measures of inflation fall after an increment in the interest rate. In fact, the central bank's strong response to a specific inflation index tends to stabilize the remaining inflation measures. This co-movement explains the fact that regions of determinacy and E-stability related to different inflation measures are the same and indicates that what is relevant for determinacy and learnability is the central bank's ability to respond strongly to movements in some nominal anchor.

As the right column of Fig. 1 illustrates, if the monetary policy rule features IRS, the central bank's response can be less aggressive than a short-lived response in a monetary policy rule without persistence. A monetary policy rule with IRS signals more future hikes in the interest rate to fight a surge in inflation expectations. In fact, IRS can achieve the same variation in the output gap needed to stabilize inflation without a substantial contemporaneous rise in the interest rate by managing expectations about the future path of this variable. This effect enlarges the region of determinacy and E-stability for rules with persistent interest rate movements.

Fig. 2 shows that openness enlarges the area of determinacy and learnability since, after a hike in the interest rate, the expenditure-switching effect from domestic to foreign goods is more pronounced if the degree of openness is higher. In these cases, the central bank does not need to be so aggressive because the stronger expenditure-switching effect catalyzes the fall in the output gap.

Fig. 3 documents the contribution of nominal wage rigidities to the enlargement of the region of determinacy and learnability. Indeed, under rigid prices, more rigid nominal wages lead to inflexible real wages. Since real wages cannot adjust easily, the
reduction in domestic demand, caused by the expenditure-switching effect, exacerbates the drop in the output gap. In this situation, the central bank can respond less aggressively, engendering moderate hikes in the interest rate.

In short, the direct exchange rate channel enlarges the area of determinacy and learnability if the central bank follows contemporaneous data rules.

The findings so far suggest the following policy recommendations. First, monetary policy should be implemented with some inertia, smoothing out changes in the interest rate. Second, the most important aspect in designing an inflation targeting regime is not which index reflects better overall inflation but the commitment of the central bank to raise interest rates aggressively if necessary. Indeed, according to the preceding paragraphs, as long as the central bank responds with sufficient aggressiveness to a contemporaneous measure of inflation, the specification of such measure is irrelevant.

3.2.2. Forecast-based data rules

Figs. 4–6 show that the regions of determinacy and E-stability are the same for domestic and wage inflation and these regions differ from the areas related to rules that react to CPI inflation. Except for the closed economy case, these figures indicate that the rule that responds to CPI inflation leads to the smallest region of determinacy and E-stability compared with the specifications that react to alternative inflation measures.

Fig. 4 points to the fact that a monetary policy rule featuring IRS enlarges the area of determinacy and E-stability. Furthermore, Figs. 5 and 6 show that a high degree of trade openness shrinks the region of determinacy and E-stability and an increase in nominal wage rigidity has the opposite effect and enlarges this region.
Besides, indeterminate equilibria can be E-stable irrespective of the monetary policy rule considered, the exception being the closed economy specification. In addition, determinate equilibria can be E-unstable if the rule responds to CPI inflation and IRS is present.

Though Fig. 6 illustrates that models with rigid nominal wages show wider determinate and E-stable areas, compared with small-open-economy models in which price rigidity is the only nominal friction, the introduction of nominal wage rigidity does not qualitatively alter the fact that trade openness shrinks the areas associated with determinate and E-stable equilibria. One can see this fact by inspecting the left columns of Figs. 4 and 5, which show models with the same amount of nominal wage rigidity but different degrees of openness.

As shown in Fig. 6, incorporating nominal wage rigidity does not change the fact that forecast-based rules that respond to CPI inflation impose substantial restrictions on determinacy and E-stability conditions.

Next, I relate Figs. 4–6 to the existing literature. The right column of Fig. 5 considers the benchmark rule in a closed-economy model studied by Best (2015). This figure does not agree with the findings described in subsection 3.2 of this preceding paper. The discrepancy lies on the calibration of \( \sigma \), which measures the degree of relative risk aversion. By setting \( \sigma = 0.26 \), as she originally did, I reproduce the same areas of determinacy and learnability described in her paper. In the right column of Fig. 6, the results concerning domestic and CPI inflation have been reported in Llosa and Tuesta (2008) and in Linnemann and Schabert (2006).

If the central bank responds to current inflation, the relevant exchange rate movement is the contemporaneous appreciation after an increment in the interest rate. On the other hand, under forecast-based rules, the expected future depreciation due to the uncovered interest rate parity condition determines future expected inflation.
In fact, the future depreciation, triggered by a hike in the interest rate, signals an expected expenditure switching from foreign goods to domestic goods, leading to increased expected domestic demand. This demand pressure induces expected higher domestic prices and a rising expected output gap, which also increase expected CPI and wage inflation. This chain of events validates any extrinsic sudden surge of inflationary expectations.

In short, the central bank’s aggressive response causes a high interest rate differential relative to the foreign interest rate, leading to a high expected future depreciation, which exacerbates future expected inflation. Therefore, monetary policy activism and openness are related to indeterminate and E-unstable equilibria. One can see in Fig. 4 that restricting active movements in the interest rate is consistent with areas of determinacy and E-stability since in this areas there is an upper bound on $\phi_y$ (domestic and wage inflation) or on $\phi_p$ (CPI inflation).

For any inflation measure, Fig. 5 shows that increasing the degree of openness shrinks the area of determinacy and learnability because the expected expenditure switching effect is stronger if the economy exposes itself more to trade.

Since the high expected future depreciation directly affects expected CPI, the exacerbated increase in expected CPI inflation is stronger compared with alternative inflation measures. Hence, as described in Fig. 4, responding to CPI inflation causes substantial reductions in determinate and E-stable areas compared with rules that respond to domestic or wage inflation. These reductions become more considerable with an increase in the degree of openness, as depicted in the left column of Fig. 5.

Fig. 6 shows that an increase in nominal wage rigidity enlarges the regions of determinacy and learnability as in the case of contemporaneous monetary policy rules. Thus, the implications of the direct exchange rate channel for the size of the areas of determinacy and learnability, in the case of forecast-based data rules, depend on the relative strength between trade openness and the degree of nominal rigidity.

In general, central banks respond to inflation forecasts because this reaction takes into consideration the transmission lags of monetary policy. In this case, as a policy recommendation based on the results of this paper, central banks need to take into...

![Fig. 4. Forecast-based data rule: baseline calibration.](image-url)
account the degree of openness in deciding a particular inflation measure to target. In fact, CPI inflation is adequate as a target for moderate degrees of trade openness, but very open economies should focus on domestic inflation. Besides, monetary authorities should always smooth out movements in interest rates.

4. Speed of convergence

4.1. Methodology

A measure of the speed of convergence, as proposed in Christev and Slobodyan (2014), uses the real parts of the eigenvalues of the derivative matrices $D_{\tau}(\pi, \xi) - I$, $D_{\tau}(\xi) - I$ and $D_{\tau}(\xi, c) - I$. In my notation, $\text{Re}(\lambda_i)$ represents each one of them.

The conditions that guarantee E-stability are: all real parts of the eigenvalues of the derivative matrices above are negative. But even if the equilibrium satisfies E-stability conditions, there are situations in which, according to their magnitudes, eigenvalues with negative real parts may lead to fast or slow convergence to a REE.

12 In fact, establishing these conditions is equivalent to showing that all real parts of the eigenvalues of the Jacobian of the T-map are less than 1.
If the derivative matrices have \( p \) eigenvalues \( \lambda_i \), each one with a negative real part \( \operatorname{Re}(\lambda_i) \), the following expression characterizes the measure for the speed of convergence \( S \):

\[
S = \min \left\{ |\operatorname{Re}(\lambda_1)|, |\operatorname{Re}(\lambda_2)|, \ldots, |\operatorname{Re}(\lambda_p)| \right\}
\]

where \( |\operatorname{Re}(\lambda_i)| \) is the absolute value of \( \operatorname{Re}(\lambda_i) \).

Indeed, this measure recognizes that, in the long run, the eigenvalue satisfying E-stability conditions with the smallest absolute magnitude controls the dynamics governing the convergence to a REE.

4.2. Results

I report the median of the speed of convergence across simulations under alternative specifications for the interest rate rule. I focus on the overall performance of each combination of rule, calibration and inflation measure. For this reason, instead of highlighting specific calibrations yielding faster convergence, I choose to report the median speed across simulations.

I investigate a benchmark rule under three calibrations: the baseline calibration (Base) described in subsection 2.3, a closed economy (Closed) in which \( \alpha = 0 \), and an economy with price rigidity only (Price), in which \( \theta_w = 0 \). In addition, under the baseline calibration, I study an interest rate smoothing (IRS) rule in which \( \phi_r = 0.65 \).

Since E-stable equilibria are always determinate, for contemporaneous data rules, the speed of convergence cannot be a metric to evaluate REE driven by sunspots. Though, for the sake of brevity, there is no table reporting results for these rules, I briefly summarize the main findings. First, the magnitudes of the speed of convergence under alternative inflation measures are virtually
the same and there is no inflation index that clearly leads to faster convergence. Second, the presence of the direct exchange rate channel slows down the convergence to the REE because increasing degrees of openness or nominal wage rigidity always reduce the speed of convergence. Finally, a rule featuring IRS leads to faster convergence compared with the benchmark rule.\(^{13}\)

I concentrate the analysis on forecast-based data rules because for these rules, in some specifications, learnable equilibria can be determinate or indeterminate. I use the speed of convergence as an additional criterion to select among learnable equilibria, by comparing the magnitudes of $S$ for determinate and indeterminate equilibria that are E-stable.

Tables 1 and 2 report the speed of convergence for learnable equilibria related to forecast-based data rules.\(^{14}\) The first table concerns determinate and E-stable equilibria while the second relates to indeterminate and E-stable equilibria. In each table, the first column shows the inflation measures under investigation, which are domestic inflation ($\pi_{H,t}$), CPI inflation ($\pi_{CPI,t}$), and nominal wage inflation ($\pi_{W,t}$). For the closed economy calibration, E-stable equilibria are always determinate. Hence, Table 2 does not display a column associated with this calibration.

For a given combination of rule, calibration and inflation measure, Tables 1 and 2 indicate that, in general, the median $S$ is bigger for determinate equilibria than for indeterminate REE; the exception being the benchmark rule responding to CPI inflation under the baseline calibration. If one uses the speed of convergence as an additional criterion to select among learnable equilibria, for the majority of specifications, Tables 1 and 2 show that the relevant region of E-stable equilibria comprises also determinate equilibria because, in these cases, the dynamic system converges faster to them.

In contrast, for the benchmark rule responding to CPI under the baseline calibration, determinate equilibria are no longer selected as the fast learnable equilibria. In this situation, the relevant area of learnable equilibria may include equilibria driven by sunspots. Thus, responding to CPI inflation may induce this undesirable feature in learnable equilibria selected according to the speed of convergence. Moreover, responding to CPI inflation also brings about the smallest speed of convergence for a given configuration of monetary policy rule and calibration. For this reason, responding to domestic or wage inflation always leads to equilibria with good properties if one uses the speed of convergence as a selection device among learnable equilibria.

Next, I describe in detail the results in Table 1 and contrast them to the findings reported in Table 2.

For each rule in Table 1, the magnitudes of the speed of convergence regarding the specifications that respond to $\pi_{H,t}$ and $\pi_{W,t}$ are very close. However, in open economies, forecast-based rules that respond to CPI inflation ($\pi_{CPI,t}$) yield slower convergence to the REE. From the perspective of the speed of convergence, $\pi_{H,t}$ and $\pi_{W,t}$ clearly dominate $\pi_{CPI,t}$. On the other hand, there is no clear ranking between $\pi_{H,t}$ and $\pi_{W,t}$.

For non-CPI inflation measures, according to columns 2–4, more open economies lead to a mild increase in the speed of convergence and introducing nominal wage rigidity reduces the speed of convergence. In addition, a comparison between column 5 and the benchmark (column 2) shows that the rule with IRS slows down convergence relative to the benchmark.

For rules that respond to CPI inflation, according to columns 2–4, increasing openness and introducing nominal wage rigidity reduce the speed of convergence. Moreover, by comparing column 5 to the benchmark (column 2), one can see that the rule that responds to $\pi_{CPI,t}$ and features IRS leads to faster convergence.

In Table 2, compared with the benchmark, the rule with IRS leads to faster convergence irrespective of the inflation measure. Besides, as in Table 1, non-CPI measures dominate $\pi_{CPI,t}$ and more rigid nominal wages slow down convergence.

In short, the magnitudes of the speed of convergence in Tables 1 and 2 differ, though forecast-based rules that respond to CPI inflation invariably lead to the smallest speed of convergence for a given combination of monetary policy rule and calibration. Moreover, the speed of convergence criterion always select learnable equilibria that are determinate if the rule responds to domestic or wage inflation.

### 5. Conclusion

In a small-open economy model featuring both price and nominal wage rigidities, I investigated the determinacy and learnability conditions of REE under a handful of possible monetary policy rules. Moreover, I analyzed the speed of learning under these rules, adding another metric through which the desirability of a given rule should be gauged.

Indeed, from the perspective of determinacy and learnability of rational expectations equilibrium (REE), this paper suggests that the rule that responds to CPI inflation does not provide any improvement in the central bank’s ability to promote the

---

\(^{13}\) The magnitudes of $S$ are 0.021 (Base), 0.032 (Closed), 0.097 (Price) and 0.025 (IRS).

\(^{14}\) In the column Closed, the results for $\pi_{CPI,t}$ and $\pi_{CPI,t}$ are the same, since in a closed economy there is no difference between domestic and CPI inflation. In the column Price, the nominal wage is absent since the dynamic system describing the equilibrium does not include this variable.
convergence of an economy to a determinate and learnable REE, nor does it lead to a faster convergence when compared with rules that react to contending inflation measures.

Acknowledgments

Financial support from the Brazilian Council of Science and Technology (CNPq) is gratefully acknowledged. I am also grateful to Hamid Beladi, the editor, and an anonymous referee for helpful comments and suggestions. The views expressed in this paper are my own and should not be interpreted as representing the positions of the Central Bank of Brazil or its board members.

Appendix A. A small open economy model with sticky wages and prices

The small open economy consists of a continuum of households and firms indexed by \( h \) and \( j \), both belonging to the interval \([0,1]\). The model abstracts from capital accumulation and features wage and price stickiness.

The world economy comprises a continuum of small open economies indexed by \( i \) in the interval \([0,1]\). These economies share identical preferences, technology and market structure. Since each economy is of measure zero, its domestic policy decisions do not affect the remaining countries.

• **Households and wage-setting behavior**

  The representative household \( h \) maximizes the expected flow of utility given by the expression:

  \[
  E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\alpha}}{1-\alpha} - \frac{N_t(h)^{1+\psi}}{1+\psi} \right]
  \]

  The parameter \( \beta \), restricted to be in the unity interval, is the discount factor and \( \alpha \) measures the degree of relative risk aversion. Finally, the parameter \( \psi \), a positive number, is the inverse of the Frisch labor supply elasticity. The variable \( C_t \) stands for aggregate consumption and \( N_t(h) \) denotes labor supply.

  In fact, \( C_t \) is an index that combines bundles of domestic and imported goods according to the expression \( C_t = [(1-\alpha)C_{H_t} + \alpha^tC_{F_t}]^{\frac{1}{1-\alpha}} \), where \( C_{H_t} \) is an index of consumption of domestic goods and \( C_{F_t} \) is an index of imported goods. The parameter \( \alpha \) measures the share of domestic consumption allocated to imported goods and can be interpreted as an index of openness; the parameter \( \eta \) measures the substitutability between domestic and foreign goods.

  The following functions define the indices of consumption \( C_{H_t} \) and \( C_{F_t} \): \( C_{H_t} = (\int_0^1 C_{H_t}(j)^{1-\gamma} dj)^{\frac{1}{1-\gamma}} \) and \( C_{F_t} = (\int_0^1 C_{F_t}(j)^{1-\gamma} dj)^{\frac{1}{1-\gamma}} \).

  The parameter \( \gamma \) elasticity of substitution between types of goods produced in the home country, which are indexed by \( j \), while \( \eta \) measures the substitutability between goods produced in different foreign economies.

  Here, \( C_{H_t} \) is the aggregate quantity of goods imported from country \( i \) and consumed domestically. By symmetry, since all economies share the same structure, the expression for \( C_{F_t} \) is: \( C_{F_t} = (\int_0^1 C_{F_t}(j)^{1-\gamma} dj)^{\frac{1}{1-\gamma}} \).

  The variable \( C_{H_t}(j) \) stands for types of goods produced in a given country \( i \), which are also indexed by \( j \).

  In each period the household faces the budget constraint given by the equation:

  \[
  \int_0^1 P_{H_t}(j)(H_t(j)jdj \leq D_t + W_t(h)N_t(h) + T_t
  \]

  The symbol \( P_{H_t}(j) \) represents the price of domestic good \( j \) and \( P_{t,i}(j) \) is the price of variety \( j \) imported from country \( i \) in domestic currency.

  The representative domestic household receives lump-sum net transfers \( T_t \) and labor income \( W_t(h)N_t(h) \), where \( W_t(h) \) stands for nominal wages. The variable \( D_t \) is the payoff in \( t \) of the portfolio held in \( t-1 \) and \( \Psi_{t,t+1} \) denotes the stochastic discount factor for one-period ahead nominal payoffs.

**Table 2**

Speed of convergence for indeterminate equilibria forecast-based data rules.

<table>
<thead>
<tr>
<th>Index</th>
<th>Base</th>
<th>Price</th>
<th>IRS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_{H,t} )</td>
<td>0.017234</td>
<td>0.192231</td>
<td>0.018516</td>
</tr>
<tr>
<td>( \pi_{F,t} )</td>
<td>0.012561</td>
<td>0.097692</td>
<td>0.015730</td>
</tr>
<tr>
<td>( \pi_{H,t} )</td>
<td>0.017318</td>
<td>–</td>
<td>0.018640</td>
</tr>
</tbody>
</table>

Note: Baseline (Base), closed economy (Closed), price.

Rigidity only (Price) and Interest Rate Smoothing (IRS).
natural level of macroeconomic variables.

Firms and price-setting behavior

The production function \( N_t = \int_{0}^{\infty} f_{t \lambda}(j) \frac{1}{1 - \gamma} dj \).

Following Erceg et al. (2000), in each period, only a fraction \((1 - \theta_{w})\) of households can reset wages optimally in order to maximize the expected flow of utility.

Next, I discuss the wage decision. The representative household \( h \) supplies differentiated labor inputs \( N_t(j, h) \) for the production of each variety \( j \). Indeed, total labor supplied and the aggregate wage index are given by \( N_t(h) = \int_{0}^{1} N_t(j, h) dj \) and \( W_t = \left( \int_{0}^{1} W_t(h) \right)^{1 - \epsilon_{w}} dh \).

Following Erceg et al. (2000), in each period, only a fraction \((1 - \theta_{w})\) of households can reset wages optimally in order to maximize the expected flow of utility. The log-linear rule that approximates the optimal wage-setting strategy is the following:

\[
W_t = \frac{1 - \beta \theta_{w}^{\infty}}{1 + \beta \phi \epsilon_{w}} \sum_{k=0}^{\infty} \left( \beta \phi \right)^{k} E_t \left[ \mu_{w} + \mu_{s_{1-k}} + \phi \epsilon_{w} W_{t-1-k} + P_{t-1-k} \right]
\]

The symbol \( W_t \) denotes the log of the newly set nominal wage \( W_t \), the log of the steady-state wage mark-up is \( \mu_{w} = \log(\omega_{w}/\tau) \), \( m_{s} \) stands for the marginal rate of substitution between consumption and labor in log, \( w_{t} \) is the log of nominal wages and \( p_{t} \) represents the log of the consumer price index.

Under the assumed wage-setting scheme, the aggregate wage index evolves according to the equation:

\[
W_t = \theta_{w} W_{t-1}^{1 - \epsilon_{w}} + (1 - \theta_{w})(W_{t-1})^{1 - \epsilon_{w}}
\]

The log-linearization around the steady state yields the following formula for the wage inflation: \( \pi_{W_t} = (1 - \theta_{w})(W_t - W_{t-1}) \).

After some algebra, the combination of the expressions describing the optimal wage-setting strategy and the evolution of the aggregate wage index results in Eq. (3) of the main text.

• Firms and price-setting behavior

The production function \( Y_t(j) = A_{t} N_t(j) \) describes the technology for firm \( j \). The variables \( Y_t(j) \) and \( N_t(j) \) represent output and an index of labor input used by firm \( j \). The technology shock is \( A_{t} \), with \( a_{t} = \log(A_{t}) \) following a first order autoregressive process.

The expression \( N_t(j) = \int_{0}^{1} N_t(j, h) \right) \mu_{w}^{\infty} dh \) defines the index of labor input \( N_t(j) \) and the aggregate output is given by \( Y_t = \left( \int_{0}^{1} Y_t(j) \right)^{\mu_{w}^{\infty} dh} \), where \( \epsilon_{w} \) is the elasticity of substitution between labor varieties and \( \epsilon \) is the elasticity of substitution across different good varieties.

Firms operate in a monopolistic competitive market and set prices in staggered fashion using the scheme proposed by Calvo (1983), in which only a fraction \((1 - \theta_{p})\) of firms can adjust prices. In this context, in each period, these firms reset their prices to maximize expected profits. The following log-linear rule approximates the optimal price-setting strategy:

\[
P_{h_{t}} = \mu^{p h} + (1 - \beta \theta_{p}^{\infty}) \sum_{k=0}^{\infty} \left( \beta \theta_{p}^{\infty} \right)^{k} E_t \left[ m_{c_{t-k}} + P_{h_{t-1-k}} \right]
\]

The variable \( P_{h_{t}} \) represents the newly set domestic prices \( P_{h_{t}} \). In log, \( \mu^{p h} = \log(\tau) \) is the log of the steady-state markup, \( m_{c} \) stands for the log of real marginal costs, and \( P_{h_{t}} \) denotes the log of the domestic price index \( P_{h_{t}} \).

Real marginal costs in log scale are given by \( m_{c} = -v + w_{t} - P_{h_{t}} - a_{t} \), where \( w_{t} \) is the log of nominal wages and \( v = \log(1 - \tau) \), with \( \tau \) representing an employment subsidy. This subsidy neutralizes the distortion due to firms’ market power, leaving the economy with nominal stickiness as the only effective distortion. Hence, the flexible price equilibrium is efficient. This equilibrium defines the natural level of macroeconomic variables.
Under the assumed price-setting behavior, the equation below describes the dynamics of the domestic price index:

\[ p_{ht} = \left[ \theta_{ph} p_{ht-1}^{1-\epsilon} + \left( 1 - \theta_{ph} \right) \left( P_{ht}^{-1} \right)^{1-\epsilon} \right]^{1/\epsilon}. \]

The log-linearization around the steady state yields the formula involving domestic inflation: \( \pi_{ht} = (1 - \theta_{ph}) (P_{ht}^{-1} - p_{ht-1}) \). After some algebra, the expressions describing the optimal price-setting strategy and the dynamics of the domestic price index lead to the new Keynesian Phillips curve as stated by Eq. (2) of the main text.

**Market clearing conditions and monetary policy**

The domestic good market clearing condition yields the following expression:

\[ Y_{ht}(j) = C_{ht}(j) + \int_{0}^{1} C_{ht}(j) di \]

After some algebra involving the definitions of the demand functions for \( C_{ht}(j) \) and \( C_{ht}(j) \), I have

\[ Y_{ht}(j) = \left( \frac{P_{ht}(j)}{P_{ht}} \right)^{-\epsilon} \left[ (1 - \alpha) \left( \frac{P_{ht}}{P_{i}} \right)^{-\eta} C_{i} + \alpha \int_{0}^{1} \left( \frac{P_{ht}}{P_{i}} \right)^{-\eta} C_{i} di \right] \]

The variable \( C_{ht}(j) \) denotes the demand from country \( i \) of good \( j \) produced in the home economy, \( \Xi_{ht} \) is the nominal exchange rate, and \( P_{ht} \) is the price index for goods imported by country \( i \) expressed in its own currency. Lastly, \( P_{i} \) is the consumer price index for households living in country \( i \).

Using the definition of aggregate output \( Y_{t} = (\int_{0}^{1} Y_{t}(j)^{1/\epsilon} dj)^{\epsilon} \), I get the following expression:

\[ Y_{t} = \left( \frac{P_{ht}}{P_{i}} \right)^{-\eta} C_{i}(1 - \alpha) + \alpha \int_{0}^{1} (S_{i} S_{ht})^{-\eta} Q_{ht}^{1-\eta} di \]

The effective terms of trade for country \( i \) is \( S_{i} = \frac{\Xi_{ht} P_{i}}{P_{ht}} \) and \( S_{ht} = \frac{P_{ht}}{P_{i}} \) denotes the bilateral terms of trade between country \( i \) and the domestic economy \( H \). Finally, \( Q_{ht} = \frac{E_{ht} P_{ht}}{P_{i}} \) represents the bilateral real exchange rate between countries \( i \) and \( H \).

The labor market clearing condition is:

\[ N_{t} = \int_{0}^{1} N_{t}(j) dj = \int_{0}^{1} N_{t}(h) dh \]

To close the model, I assume that the central bank follows the interest rate rules described in subsection 2.2 of the main text.

**Appendix B. The matrix representation of the parsimonious system**

Concerning the private sector equilibrium conditions, I reduce the original system in subsection 2.1 to a smaller one with five endogenous variables: \( \dot{y}_{t}, \pi_{ht}, \pi_{wt}, w_{t}, \) and \( r_{t} \). To accomplish this reduction, I use Eq. (6) to solve for \( s_{t} \) and Eq. (5) to get the following expression for CPI inflation: \( \pi_{CPI} = \pi_{ht} + \alpha_{\sigma_{yt}} (y_{t} - \dot{y}_{t-1}) + \alpha_{\sigma_{yt}} \Delta s_{yt} \), where \( \Delta s_{yt} = \sigma_{yt}(\Delta y_{t} - \dot{y}_{t}) \). After substituting out \( \pi_{CPI} \) in Eq. (4), this expression becomes Eq. (12). The parsimonious system comprises the following equations:

\[ \dot{y}_{t} = E_{t} (\dot{y}_{t} + 1) - \frac{1}{\sigma_{\alpha}} \left[ r_{t} - E_{t} (\pi_{ht-1}) \right] + \frac{1}{\sigma_{\alpha}} \bar{r}_{t} \]

\[ \pi_{ht} = \beta E_{t} (\pi_{ht-1}) + \kappa_{ph} \dot{y}_{t} + \lambda_{ph} w_{t} \]

\[ \pi_{wt} = \beta E_{t} (\pi_{wt-1}) + \kappa_{w} \dot{y}_{t} - \lambda_{w} w_{t} \]

\[ \dot{w}_{t} = \ddot{w}_{t-1} + \pi_{wt} - \pi_{ht} - \alpha \sigma_{yt} (\dot{y}_{t} - \dot{y}_{t-1}) - \alpha \Delta s_{yt} - \Delta w_{t} \]

The last equation that closes the system is one of the interest rate rules discussed in subsection 2.2. **Subsection 3.1** considers the following system of linear stochastic difference equations:

\[ x_{t} = BE_{t} x_{t+1} + Dx_{t-1} + Kv_{t} \]

\[ v_{t} = Rv_{t-1} + \xi_{t} \]

where \( x_{t} \) is a \( m \times 1 \) vector of endogenous variables and \( v_{t} \) is a \( k \times 1 \) vector of exogenous disturbances.
The compact form of the system above, which I use for determinacy analysis, is

\[ E_t z_{t+1} = J_1 z_t + J_2 v_t \]

where \( J_1 \) and \( J_2 \) are functions of the matrices \( B, D \) and \( K \), with the vector \( z_t \) defined as follows:

\[ z_t = [x_t x_{t-1}]' \]

To obtain the matrices \( B, D, K, J_1, \) and \( J_2 \), I represent the system (9) to (12) supplemented by the interest rate rule (7) or (8) as

\[ A_0 x_t = A_1 E_t x_{t+1} + A_2 x_{t-1} + A_3 v_t \]

\[ v_t = R v_{t-1} + \xi_t \]

where \( \xi_t = [y_t \pi_{H,t} \pi_{W,t} w_t r_t]' \) and \( v_t = [r_t \Delta \pi_t \Delta w_t]' \).

The 3×1 random vector \( \xi_t \) is independent and identically distributed with mean zero and variance–covariance matrix \( \Sigma \).

The matrices \( A_0, A_1, \) and \( A_2 \) are 5×5; \( A_3 \) is 5×3 and \( R \) is 3×3.

If the inverse matrix \( A_0^{-1} \) exists, expressions relating \( B, D, \) and \( K \) to the primitive matrices in system (13) are

\[ B = A_0^{-1} A_1, \quad D = A_0^{-1} A_2 \quad \text{and} \quad K = A_0^{-1} A_3. \]

For the compact form, the expressions for matrices \( J_1 \) and \( J_2 \) are

\[ J_1 = \begin{bmatrix} B & 0_{5 \times 5} & 0_{5 \times 5} & -D \end{bmatrix} \quad \text{and} \quad J_2 = \begin{bmatrix} B & 0_{5 \times 5} \end{bmatrix} \begin{bmatrix} -K \\ 0_{5 \times 3} \end{bmatrix}. \]

Next, I explicitly display the matrices in system (13) that represent the set of Eqs. (9) to (12) plus the interest rate rule (7) or (8). To be concise, I consider generalized interest rate rules, in which the central bank responds to all measures of inflation. For gauging the merits of a particular measure, I set the coefficient \( \phi_1 \).

For any rule, the definition of matrix \( R \) is

\[ R = \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \]

For each type of rule, I show next a particular set of matrices \( A_0, A_1, A_2, \) and \( A_3 \).

- The matrices for the generalized contemporaneous data rule are

\[ A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{1}{\sigma_x^2} \\ -\kappa_{ph} & 1 & 0 & -\lambda_{ph} & 0 \\ -\kappa_{w} & 0 & 1 & \lambda_{w} & 0 \\ \phi_y + \phi_{CP} \alpha \sigma_x & \phi_{CP} + \phi_H & \phi_W & 0 & -1 \end{bmatrix} \]

\[ A_1 = \begin{bmatrix} 1 & \frac{1}{\sigma_x^2} & 0 & 0 & 0 \\ 0 & \beta^2 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \alpha \sigma_x & 0 & 0 & 1 & 0 \\ \phi_{CP} \alpha \sigma_x & 0 & 0 & 0 & -\phi_r \end{bmatrix} \]


\[
A_3 = \begin{bmatrix}
\frac{1}{\sigma_\alpha} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -\alpha & -1 \\
0 & -\phi_{CP} & 0 \\
\end{bmatrix}
\]

- The matrices for the generalized forecast-based data rule are

\[
A_0 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
-\kappa_{ph} & 1 & 0 & -\lambda_{ph} & 0 \\
-\kappa_{pW} & 0 & 1 & \lambda_{pW} & 0 \\
\alpha \sigma_\alpha & 1 & -1 & 1 & 0 \\
\phi_{CP} \alpha \sigma_\alpha & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[
A_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma_\alpha} & \beta & 0 & 0 \\
0 & 0 & \beta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\phi_y + \phi_{CP} \alpha \sigma_\alpha & \phi_{CP} + \phi_H & \phi_W & 0 & 0 \\
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\alpha \sigma_\alpha & 0 & 0 & 1 & 0 \\
\phi_y & 0 & 0 & 0 & \phi_y \\
\end{bmatrix}
\]

\[
A_3 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -\alpha & -1 \\
0 & -\phi_{CP} & 0 \\
\end{bmatrix}
\]

References


